Assign 1 Fundamental Counting Principles

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Name _____ Date _____

1. A restaurant offers 4 different soft drink flavours, 5 different sandwiches and 3 different dessert selections. In how many ways can a person select one item from each category (a drink, a sandwich and a dessert)?

$4 \times 5 \times 3 = 60$

2. How many 2-digit whole numbers are there with the units digit being a multiple of 5?

$9 \times 2 = 18$. NOTE: ZERO IS A MULTIPLE OF 5.

1,2,3,4,5,6,7,8,9 — 0,5.

3. Using the digits 2, 3, 5, and 9, how many 2-digit whole numbers can be formed if repetitions are not permitted?

$4 \times 3 = 12$

4 options — 3 options left.

4. Using the digits 1, 4, 6, and 9, how many 2-digit whole numbers can be formed if repetitions are not permitted?

$4 \times 3 = 12$.

[NOTE: IF REPETITIONS ARE PERMITTED THEN $4 \times 4 = 16$]

5. A multiple-choice test has 8 questions, with 4 possible answers for each question. If a student were to guess the answer to each question, how many different ways would there b e to answer the test?

$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^8 = 2^{16}$ //.

6. A car licence plate consists of 6 characters. The first 3 characters are letters excluding I, O, Q, and U. The last 3 characters are any of the numerals from 0 to 9. How many different licence plates are possible?

$22 \times 22 \times 22 \times 10 \times 10 \times 10 = 22^3 \times 1000 = 10{,}648{,}000$

7. The dial on a combination lock contains markings which represent the numbers from 0 to 59. How many 3-number combinations are possible if the first and the third numbers must be different multiples of three, and the second number must not be a multiple of three?

From 0–59, how many multiples of 3 are there?

$3(0), 3(1), 3(2) \ldots\ldots 3(19) \rightarrow$ 20 numbers.

How many numbers are not multiples of 3: $60 - 20 = 40$

$20 \times 40 \times 19 = 15200$

8. How many different outfits consisting of a shirt, a pair of pants, and a sweater can be chosen from 4 shirts, 3 pants, and 2 sweaters?

$4 \times 3 \times 2 = 24$

9. In how many different ways can the letters in the word PRICE be scrambled?

$5 \times 4 \times 3 \times 2 \times 1 = 120$

10. In how many ways can 5 books be arranged on a shelf if 2 of the books must remain together?

- LET THE BOOKS BE [AB] C D E → $4 \times 3 \times 2 \times 1 \times 2 = 48$
- TREAT [AB] AS A BOOK, [AB] or [BA]

11. How many three-digit numbers can be formed from the digits 1, 2, 3, 4, 5 if the 2 may be used any number of times but the other digits may be used at most once in any three-digit number?

① 3 - 2's : ①

② 2 - 2's : $\underline{2}\,\underline{2}\,\underline{\times}$, $\underline{2}\,\underline{\times}\,\underline{2}$, $\underline{\times}\,\underline{2}\,\underline{2}$ } $3 \times 4 = 12$.

③ 1 or No 2's: $5 \times 4 \times 3 = 60$

ALTOGETHER: $60 + 12 + 1 = 73$ //

12. What is the sum of all possible three-digit numbers that can be written using the digits 3, 4, and 6 once in each number?

346
364
463
436
634
643

$26 \times 100 + 26 \times 10 + 26$
$2600 + 260 + 26$

2600
260
26
2886 //

13. How many different three-letter sets of initials are possible using the letters of the alphabet?

- IF THE LETTERS CAN ALL BE THE SAME: $26 \times 26 \times 26 = 26^3$
- IF THE LETTERS CANNOT BE THE SAME $26 \times 25 \times 24$

14. What is the largest number of mailboxes needed to hold 45 letters if each mailbox contains at least one letter, and no two mailboxes contain the same number of letters?

$1 + 2 + 3 + 4 + 5 + \cdots n = \frac{n(n+1)}{2} = 45.$

$n(n+1) = 90$

$n = 9$

15. How many 3-digit numbers can be formed using the digits 0, 1, 2, 3, 4 if no repetitions are allowed?

$4 \times 4 \times 3 = 48$

(1,2,3,4 — ZERO IS ALLOWED)

16. How many different three0digit security codes are possible using the digits 1-5, if the second digit cannot be the same as the first, and the third digit cannot be the same as the second?

$5 \times 4 \times 4 = 80$ A, B, C

(1,2,3,4,5 — CAN'T BE A — CAN'T BE B)

17. On a certain exam, you must answer 10 of 12 questions. How many different sets of questions can you choose? Express your answer as a whole number of sets.

NOTE: ORDER DOESN'T MATTER,

SO $12C_{10} = 12C_2 = \frac{12 \times 11}{1 \times 2} = 66$

18. How many different five-digit numbers can be formed using the digits 2, 3, 2, 3, 3?

$n = 5$

$a = 2$ (THERE ARE 2 TWOS) $\rightarrow \frac{n!}{a!\,b!} = \frac{5!}{3!\,2!}$

$b = 3$ (THERE ARE 3 THREES) $= 10$

19. If the digits can be used more than once, how many different even three-digit numbers can be written using the digits 1, 2, 3, 5, and 7?

$5 \times 5 \times 1 = 25$

20. How many different, positive three-digit numbers and four-digit numbers greater than 310 can be made using any three of the following eight digits without repetition: 1, 2, 2, 3, 4, 5, 6, 7?

- SINCE YOU HAVE 2 TWO'S, YOU CAN REPEAT THE TWO'S.
- 3 DIGIT NUMBERS:

$5 \times 7 \times 6$ (ONE: 2) $\}$ $210 + 5 = 215$

(3-7)

$5 \times 1 \times 1$ (TWO: 2's) $\}$

- 4 DIGIT NUMBERS

$7 \times 6 \times 5 \times 4$ (ONE: 2) $\}$ $840 + 252 = 1092$

$1 \times 1 \times 7 \times 6 \times 6$ (TWO 2's) $\}$

(6 WAYS TO PLACE THE 2's.)

TOTAL $= 215 + 1092 = 1307$

21. Using the pennies, nickels, dimes, and quarters, in how many ways can you have 30 cents in change?

Q	D	N	P
1		1	
	3		
	2	2	
	2	1	5
	1	4	
	1	3	5
	1	2	10
	1	1	15
	1	0	20

Q	D	N	P
1			5
		6	0
		5	
		4	
		3	
		2	
		1	
			30

17 ways.

22. In how many different ways can change be made for a dollar bill using coins from a collection that contains four nickels, four dimes, four quarters, and two half-dollars?

H	Q	D	N

H	Q	D	N

22. In how many different ways can change be made for a dollar bill using coins from a collection that contains four nickels, four dimes, four quarters, and two half-dollars?

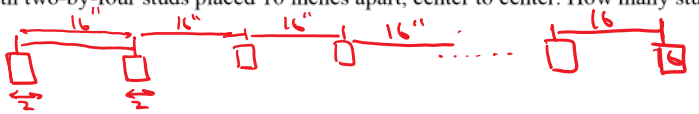
2H, 4Q, 4D, 4N.

H	Q	D	N
2			
1	2		
1	1	2	1
1	0	4	2
1	0	3	4

H	Q	D	N
	4	0	0
	3	2	1
	3	1	3
	2	4	2
	2	3	4

10 ways

23. The walls of standard houses are constructed with two-by-four studs placed 16 inches apart, center to center. How many studs are needed for a wall that is 48 feet wide?



① $48' = 48 \times 12''$
$= 4 \times 12 \times 12$
$= 576''$

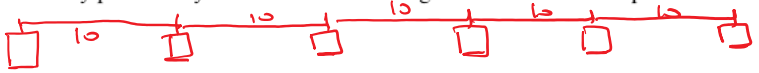
② How many 16"? $\frac{4 \times 12 \times 12}{4 \times 4} = 36$

37 STUDS

24. A holiday fruit box contains a dozen each apples, oranges, and grapefruits. What is the least number of pieces of fruit must you pick to guarantee that you have three of the same kind?

① WORST CASE SCENARIO: A, O, G, A, O, G, A → 7 FRUITS.

25. How many poles will you need to make a straight 50-foot fence with poles 10 feet apart?



6 POLES

26. A certain chocolate is packaged in a box that holds only one chocolate, a box that holds 5 chocolates, or a box that holds 25 chocolates. How many boxes are needed to fill an order for 116 chocolates if the fewest number of boxes is used and each box is filled?

~~Box~~: 1, 5, 25.

116: 25, 25, 25, 25, 5, 5, 5, 1.

8 BOXES.

27. At a party recently Chunlin and some of his friends sat in a circle and passed around a bag containing 19 hard pretzels. Each took one pretzel as the bag passed. Chunlin got the first pretzel. How many people were in the circle, including Chunlin?

19 ppl

28. Laura received in change pennies, dimes and quarters totalling $1.57. What is the minimum number of coins she could have received?

Q, Q, Q, Q, Q, Q : 1.50
P P P P P P P : 0.07
13

OR

Q Q Q Q Q , 1.25
D D D 0.30
P P : 0.02
10 ✓

29. Ed has 41 cents in change. What is the fewest number of U.S. coins he could have?

0.25 0.10 0.05 0.01 (4)

30. What is the maximum number of possible diagonals in an octagon?

$8C_2 - 8$

$\frac{8\times7}{1\times2} - 8 = 28 - 8 = (20)$

31. Suppose 200 players enter a tennis tournament. If a player is eliminated when he or she loses a match, how many matches must be played to determine a winner?

1 GAME ELIMINATES 1 PLAYER

199 GAMES ELIMINATES 199 PLAYERS.

32. What is the 50th odd whole number?

1, 3, 5, 7, 9, ... n.

$\frac{9+1}{2} = 5$ $\frac{7+1}{2} = 4$ $\frac{n+1}{2} = 50$

$n+1 = 100$

$n = 99$

33. A book contains 250 pages. How many times is the digit 2 used in numbering the pages?

① 1–10 ② 11–20 ③ 21–30 ④ 31–40 ⑤ 41–50 ⑥ 51–60 ⑦ 61–70 ⑧ 71–80 ⑨ 81–90 ⑩ 91–100 } 20

100–199 (20)

200–210 (12)

211–220 (12) ✓

221–230 (20)

231–240 (11)

241–250 (11)

TOTAL: 20+20+24+31

= 40+55

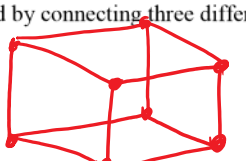
= (95).

34. In the game of *Five Flying Fish*, a goal is worth seven points and a basket is worth three points. There are no other ways to score points in this game. What is the largest score that cannot be obtained?

$3\times7 - 3 - 7$

(11).

35. How many distinct triangles can be constructed by connecting three different vertices of a cube?



$8C_3 = \frac{8\cdot7\cdot6}{1\cdot2\cdot3} = (56)$

36. In Morse code, each symbol is represented by a sequence of dashes and dots. How many distinct symbols can be represented using sequences of 1, 2, 3, or 4 dots and/or dashes?

$2\times2\times2\times2 = (16)$

37. Ten days from Thursday, it will be Sunday. What day of the week will it be 1,000,000 days from Thursday.

① Divide by 7 & Find Remainder

142857
7|1000.000
32645
1

1000,000 = 1+7K.

↑ So 1 day later would be Friday

38. Carlin wrote a 477-page book on the history of mathematics. She numbered the pages by hand, beginning with page 1. How many total digits did she write when numbering the pages?

1–9	9×1	9
10–99	90×2	180
100–477	378×3	1134
		1323

1323

39. If you use 999 digits to write page numbers consecutively starting with 1, how many page numbers could you write?

1–9	9×1
10–99	90×2
100–K	(K−100+1)3

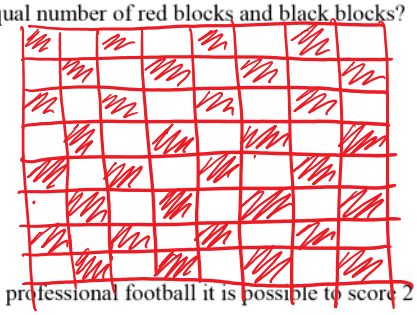
9 + 180 + 3K − 297 = 999

3K = 999 − 189 + 297

3K = 1107

K = 369

40. A standard checkerboard with 8 blocks to a side contains 204 squares of various sizes. In how many such squares are there an equal number of red blocks and black blocks?



2×2 → 7×7 = 49

4×4 → 5×5 = 25

6×6 → 3×3 = 9

8×8 → 1×1 = 1

84

41. In professional football it is possible to score 2 points (for safety), 3 points (for a field goal), or 6 points (for a touchdown). If a touchdown is scored it is possible to score one additional point. The score of a football game is 21 to 17. Find the total number of possible combinations of points to achieve this final score.

21: (6+1), (6+1), (6+1) ①

3, 3, 3, 3, 3, 3, 3 — ①

222 333 33 — $_8C_3$

222 222 333 — $_9C_3$

222 222 2223 → $_{10}C_1$

(6+1)(6+1) 322 $\frac{5!}{2!2!}$

(6+1) 322 322 $\frac{7!}{4!2!}$

(6+1) 2222222 $\frac{8!}{7!}$

17: (6+1)(6+1)(3)

3333332

3333222 2

3 222 2222

42. Charlene's calculator displays the digits 0, 1, 6, 8 and 9 so that, when the calculator is held upside-down, these digits appear to be 0, 1, 9, 8 and 6 respectively. How many three-digit numbers look the same upside-down and right –side-up when using these five possible digits? (A three-digit string that begins with either "0" or "00" is not considered a three-digit number.)

43. There are 120 five-digit numbers which use the odd digits 1, 3, 5, 7 and 9. When these numbers are arranged in numerical order from smallest to largest, in what position do 75,391 fall?

1 3,5,7,9 → 4×3×2×1 = 24

3 1,5,7,9 → 24

5 1,3,7,9 → 24

7 1 3,7,9 → 6

7 3 1,7,9 → 6

7 5 1,3,9 → 6

7 5 1,9 3 → 6

7 5 3 1 9 → 1 → 7 5 3 9 1 (1)

24 + 24 + 24 + 6 + 6 + 4

30 + 30 + 28

88th

44. In the grid shown, it is only possible to travel along an edge in the direction indicated by the arrow. What is the number of different paths from A to F: