Prep2

October 29, 2015 10:25 AM

Math Club Worksheet #2 Quadratic Problems:

1. Find all possible values of a/b if $a^2+4b^2=4ab$

$$a^2+4b^2=4ab$$
$$a^2-4ab+4b^2=0$$
$$(a-2b)^2=0$$
$$a-2b=0$$
$$a=2b$$
$$\boxed{\frac{a}{b}=2}$$

2. Let $f(x)=a^2x^2+5ax+3$ and $f(2)=2$. Find all possible values of the constant "a"

$$f(2)=a^2(4)+5(a)(2)+3=2$$
$$4a^2+10a+1=0$$
$$a=4 \quad b=10 \quad c=1$$
$$a=\frac{-10\pm\sqrt{100-4(4)}}{2(4)}$$
$$=\frac{-10\pm\sqrt{84}}{8}$$

3. Find the value of "x" if "x" is positive and $x-1$ is the reciprocal of $x+\frac{1}{2}$

① $a\left(\frac{1}{a}\right)=1$

$$(x-1)\left(x+\frac{1}{2}\right)=1$$
$$x^2-\frac{1}{2}x-\frac{1}{2}=1$$
$$x^2-\frac{1}{2}x-\frac{3}{2}=0$$
$$2x^2-x-3=0$$

$$(2x-3)(x+1)=0$$
$$\boxed{x=\frac{3}{2}} \quad x\neq-1$$

4. Let "f" be a function for which $f(x/3)=x^2+x+1$. Find the sum of all the values of "z" for which $f(3z)=7$ [amc12]

① NOTE: $f\left(\frac{x}{3}\right)=\left[3\left(\frac{x}{3}\right)\right]^2+\left[3\left(\frac{x}{3}\right)\right]+1$

$$f(n)=(3n)^2+(3n)+1$$
$$=9n^2+3n+1$$

$$f(3z)=9(3z)^2+3(3z)+1$$
$$=81z^2+9z+1$$

5. Let "a" and "b" be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + \frac{1}{b}$ and $b + \frac{1}{a}$ are the roots of the equation $x^2 - px + q = 0$. What is the value of "q"?

① $x^2 - mx + 2 = (x-a)(x-b)$

$= x^2 - (a+b)x + ab.$

$\boxed{a+b=m}$ $\boxed{ab=2}$

② $x^2 - px + q = (x - a - \frac{1}{b})(x - b - \frac{1}{a})$

$p = a + b + \frac{1}{a} + \frac{1}{b}.$ $q = (a + \frac{1}{b})(b + \frac{1}{a})$

$q = ab + \frac{1}{ab} + 2.$

$q = 2 + \frac{1}{2} + 2$

$\boxed{q = 4.5}$

6. Find all real solutions to $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$

① $a^0 = 1$

$x^2 - 9x + 20 = 0.$

$(x-4)(x-5) = 0$

$x = 4$ $x = 5.$

② $(1)^x = 1$

$x^2 - 5x + 5 = 1$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4, x = 1$

③ $(-1)^{2x} = 1$

$x^2 - 5x + 5 = -1$

$x^2 - 5x - 6 = 0.$

$(x-6)(x+1) = 0.$

$x = 6$ $x = -1.$

$6^2 - 9(6) + 20$ $(-1)^2 + 9 + 20$

EVEN! EVEN

7. Find all solutions to the system of equations:

$x^2 + yz = 39$

$x - yz = -33$

$y + z = 13$ ← 12 ???

① ALWAYS ELIMINATE ONE VARIABLE

$z = 13 - y.$

$x^2 + y(13-y) = 39.$

$x - y(13-y) = -33.$

$x^2 + x = 6.$

$x^2 + x - 6 = 0.$

$(x+3)(x-2) = 0$

$x = -3$ $x = 2.$

② $x = 2.$

$4 + yz = 39$

$yz = 35.$

$y(13-y) = 35$

$0 = y^2 - 13y + 35$

8. Find the roots of $x^2+\left(a-\frac{1}{a}\right)x-1=0$ in terms of "a"

$$(x+a)(x-\tfrac{1}{a})=0$$

$$x=-a \quad x=+\tfrac{1}{a}.$$

9. Find the solutions to $\left(x^4-11x^3+24x^2\right)-\left(4x^2-44x+96\right)=0$

① $x^2(x^2-11x+24)-4(x^2-11x+24)=0$

$(x^2-4)(x^2-11x+24)=0$

$(x+2)(x-2)(x-3)(x-8)=0$

$\downarrow$

$x=-2,\ 2,\ 3,\ 8.$ //

$\boxed{10.}$ If $\frac{x^2-bx}{ax-c}=\frac{m-1}{m+1}$ has solutions for "x" such that each solution is the negative of the other, then find "m" in terms of "a" and "b". [AHSME]

①

$(x^2-bx)(m+1)=(m-1)(ax-c)$

$\cancel{mx^2}-mbx+\cancel{x^2}-bx=amx-ax-cm+c$

$(m+1)x^2-mbx-bx-amx+ax=-cm+c$

$(m+1)x^2-(mb+b+am-a)x+cm-c=0.$

② THE solutions ARE NEGATIVE OF EACH OTHER. ∴ $r_1+r_2=0$.

sum of roots is zero.

$mb+b+am-a=0$

$m(a+b)=a-b$

$$\boxed{m=\frac{a-b}{a+b}}$$ //

11. Find constants "a" and "b" such that $b-a$ is as small a possible, and the entire graph of the equation

$y=\frac{1-x^2}{1+x^2}$ lies within $a<y\le b$

① No vertical asymptotes. $(1+x^2)$.

$y=\frac{-x^2+1}{1+x^2} = -\left(1-\frac{2}{x^2+1}\right)$

$= -\left(\frac{x^2-1}{x^2+1}\right) = -1+\frac{2}{x^2+1}$.

$= -\left(\frac{x^2+1-2}{x^2+1}\right)$

$= -\left(\frac{x^2+1}{x^2+1} - \frac{2}{x^2+1}\right)$.

② let $x=0$.

$y=-1+\frac{2}{1}$

$y=1$.

let $x=10{,}000$.

$y=-1+\frac{2}{100\ldots}$

$=-1$

$\therefore -1<y\le 1$

$\boxed{b-a=2}$

12. Prove that if $\frac{a+b}{a}=\frac{b}{a+b}$ then "a" and "b" can't both be real numbers

$(a+b)(a+b)=ab$

$a^2+2ab+b^2=ab$.

$a^2+ab+b^2=0$.

$a=1 \quad b=b \quad c=b^2$.

$a=\frac{-b\pm\sqrt{b^2-4b^2}}{2}$

$a=\frac{-b\pm\sqrt{-3b^2}}{2}$

imaginary

13. Let "m" and "n" be the roots of $ax^2+bx+c=0$. Prove that if $m^2+n^2=1$, then $2ac=b^2-\boxed{c^2}$

$\boxed{a^2\ ??}$!

①

$ax^2+bx+c=a(x-m)(x-n)$

$=a(x^2-mx-nx+mn)$

$=ax^2-(am+an)x+amn$

$= ax^2 + bx + c$

$x^2+\frac{b}{a}x+\frac{c}{a}=0=(x-m)(x-n)$

$=x^2-(m+n)x+mn$.

① $-\frac{b}{a}=m+n \qquad \frac{c}{a}=mn$.

$\frac{b^2}{a^2}=m^2+2mn+n^2$

$\frac{b^2}{a^2}=m^2+2\frac{c}{a}+n^2$

$\frac{b^2}{a^2}=1+\frac{2c}{a}$.

$\frac{b^2}{a^2}=\frac{a+2c}{a}$

$\frac{b^2}{\cancel{a^2}}=\frac{a^2+2ac}{\cancel{a^2}}$

$\boxed{b^2-a^2=2ac}$

②

$am+an=-b \qquad amn=c$.

$a(m+n)^2=-b$.

$a^2(m^2+2mn+n^2)=b^2$.

$a^2(m^2+n^2)+a\cdot 2\cdot amn=b^2$.

$a^2+2ac=b^2$