SOL HW 4.5

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Name: Key Date:

Math 9 Enriched: Section 4.5 Factoring Difference and Sums of Powers

Difference of squares: $a^2 - b^2 = (a+b)(a-b)$

Difference and Sums of Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Difference of Powers: $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \ldots + ab^{n-2} + b^{n-1})$

Difference of Sums: $a^{2n+1} + b^{2n+1} = (a+b)(a^{2n} - a^{2n-1}b + a^{2n-2}b^2 \ldots - ab^{2n-1} + b^{2n})$

1. Factor each and simplify the following expressions completely:

a) $x^6 - 64$ $(x^3+8)(x^3-8)$ $(x+2)(x^2-ab+b^2)(x-2)(x^2+ab+b^2)$	b) $9^3 - a^6x^6$ $(3^2)^3 - (ax)^6$ or $(3^3)^2 - (a^3b^3)^2$ $3^6 - (ax)^6$ $= (3^3 - a^3b^3)(3^3 + a^3b^3)$ $= (3-ab)(9-3ab+a^2b^2)(3+ab)(9+3ab+a^2b^2)$ $= (3-ax)(3^5 + 3^4ax + 3^3a^2x^2 + 3^2a^3x^3 + 3a^4x^4 + a^5x^5)$
c) $81 - (3a+2)^4$ $3^4 - (3a+2)^4$ $[3^2 + (3a+2)^2][3^2 - (3a+2)^2]$ $[9 + (9a^2+12a+4)][9 - (9a^2+12a+4)]$ $= (9a^2+12a+13)(-9a^2-3a-4)$ $-(9a^2+12a+13)(9a^2+3a+4)$	d) $\dfrac{1000+27x^3}{100-9x^2}$ $\dfrac{10^3+(3x)^3}{10^2-(3x)^2}$ $= \dfrac{\cancel{(10+3x)}(100-30x+9x^2)}{\cancel{(10+3x)}(10-3x)}$ //
e) $\dfrac{a^3-27b^3}{a^2-9b^2}$ $\dfrac{a^3-(3b)^3}{a^2-(3b)^2} = \dfrac{\cancel{(a-3b)}(a^2+3ab+9b^2)}{\cancel{(a-3b)}(a+3b)}$ //	f) $y^6 + 16y^3 + 15$ $(y^3+15)(y^3+1)$ $(y^3+15)(y+1)(y^2-y+1)$ //
g) $x^6 - 7x^3 - 8$ $(x^3-8)(x^3-1)$ $(x-2)(x^2+2x+4)(x-1)(x^2+x+1)$ //	h) $8y^6 - 9y^3 + 1$ $8 \to -1 = -8$ $1 \to -1 = -1$ $(8y^3-1)(y^3-1)$ $(2y-1)(4y^2+2y+1)(y-1)(y^2+y+1)$ //

i) $x^6 - 26x^3 - 27$	j) $27y^6 + 35y^3 + 8$

2. Factor completely: $-a^2b^2 + 2ab^3 - b^4 + a^2c^2 - 2abc^2 + b^2c^2$

$-b^2(a^2 - 2ab + b^2) + c^2(a^2 - 2ab + b^2)$

$-b^2(a-b)(a-b) + c^2(a-b)(a-b)$

$(a-b)(a-b)(c^2 - b^2)$

$(a-b)(a-b)(c+b)(c-b)$ //

3. Factor completely with integral coefficients: $x^{12} - y^{12}$

$x^{12} - y^{12}$

$= (x^6 + y^6)(x^6 - y^6)$

$= (x^2+y^2)(x^4 - x^2y^2 + y^2)(x^3 - y^3)(x^3 + y^3)$

$= (x^2+y^2)(x^4 - x^2y^2 + y^2)(x-y)(x^2 + xy + y^2)(x+y)(x^2 - xy + y^2)$ //

4. Factor and simplify the expression as much as possible: $\left(\frac{a^3-1}{a^2-1}\right)\left(\frac{a^2+2a+1}{a^3+1}\right)\left(\frac{a^2-a+1}{a+1}\right)$

5. When $x^9 - x$ is factored as completely as possible into polynomials and monomials with integral coefficients, how many factors are there?

$x^9 - x = x(\underline{x^8 - 1})$

$= x(x^4+1)(\underline{x^4 - 1})$

$= x(x^4+1)(x^2+1)(\underline{x^2 - 1})$

$= (\underline{x})(\underline{x^4+1})(\underline{x^2+1})(\underline{x+1})(\underline{x-1})$

6. If $x+y=4$ and $xy=2$, then find x^6+y^6

7. Find the value of $x^6+\frac{1}{x^6}$ if the value of $x+\frac{1}{x}=3$.

$\frac{1}{x}+x=3$

$\left(\frac{1}{x}+x\right)^2=9$

$\frac{1}{x^2}+1+1+x^2=9$

$\boxed{\frac{1}{x^2}+x^2=7}$

$\left(\frac{1}{x^2}+x^2\right)\left(\frac{1}{x}+x\right)=21$

$\frac{1}{x^3}+\frac{1}{x}+x+x^3=21$

$\frac{1}{x^3}+x^3+3=21$

$\boxed{\frac{1}{x^3}+x^3=18}$

$\left(\frac{1}{x^3}+x^3\right)^2=18^2$

$\left(\frac{1}{x^3}+x^3\right)\left(\frac{1}{x^3}+x^3\right)=324$

$\frac{1}{x^6}+1+1+x^6=324$

$\boxed{\frac{1}{x^6}+x^6=322}$

8. If $a+b=1$, $a^2+b^2=2$, find the value of a^4+b^4

$a+b=1$ $\quad a^2+b^2=2$ $\quad a^4+b^4$

$(a^2+b^2)(a^2+b^2)=4$

$a^4+2a^2b^2+b^4=4$

$(a+b)(a+b)=1$

$a^2+2ab+b^2=1$

$2ab+2=1$

$2ab=-1$

$4a^2b^2=1$

$2a^2b^2=\frac{1}{2}$

$a^4+b^4+\frac{1}{2}=4$

$\boxed{a^4+b^4=\frac{7}{2}}$

9. If $\frac{1}{a+c}=\frac{1}{a}+\frac{1}{c}$, find the value of $\left(\frac{a}{c}\right)^3$.

$\frac{1}{a+c}=\frac{1}{a}+\frac{1}{c}$ $\quad \left(\frac{a}{c}\right)^3=??$

$\frac{1}{a+c}=\frac{c}{ac}+\frac{a}{ac}$

$\frac{1}{a+c}=\frac{a+c}{ac}$

$ac=(a+c)^2$

$ac=a^2+2ac+c^2$

$(a-c)0=(a^2+ac+c^2)(a-c)$

$a^3-c^3=(a^2+ac+c^2)(a-c)$

$0=a^3-c^3$

$\frac{c^3}{c^3}=\frac{a^3}{c^3}$

$\boxed{1=\left(\frac{a}{c}\right)^3}$

$a^3=c^3$

$a\neq c$

10. Find the sum of $\frac{1}{3+2\sqrt{2}}+\frac{1}{2\sqrt{2}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}+2}+\frac{1}{2+\sqrt{3}}$.

11. Challenge: Find the sum of: $\frac{1}{\sqrt[3]{1}+\sqrt[3]{2}+\sqrt[3]{4}}+\frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}+\frac{1}{\sqrt[3]{9}+\sqrt[3]{12}+\sqrt[3]{16}}$

11. Challenge: Find the sum of: $\frac{1}{\sqrt[3]{1}+\sqrt[3]{2}+\sqrt[3]{4}}+\frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}+\frac{1}{\sqrt[3]{9}+\sqrt[3]{12}+\sqrt[3]{16}}$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$\frac{a^3-b^3}{a-b}=(a^2+ab+b^2)$

$\boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$

11. $\frac{1}{\sqrt[3]{1}+\sqrt[3]{2}+\sqrt[3]{4}} \leftarrow \frac{1}{(\sqrt[3]{1})^2+(\sqrt[3]{2})(\sqrt[3]{1})+(\sqrt[3]{2})^2} = \frac{\sqrt[3]{1}-\sqrt[3]{2}}{1-2}$ $\quad \frac{a-b}{a^3-b^3}=\frac{1}{a^2+ab+b^2}$

$\frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}} \rightarrow \frac{1}{(\sqrt[3]{2})^2+(\sqrt[3]{2})(\sqrt[3]{3})+(\sqrt[3]{3})^2} = \frac{\sqrt[3]{2}-\sqrt[3]{3}}{2-3}$

$\frac{1}{\sqrt[3]{9}+\sqrt[3]{12}+\sqrt[3]{16}} \rightarrow \frac{1}{(\sqrt[3]{3})^2+(\sqrt[3]{3})(\sqrt[3]{4})+(\sqrt[3]{4})^2} = \frac{\sqrt[3]{3}-\sqrt[3]{4}}{3-4}$

$(-\sqrt[3]{1}+\sqrt[3]{2})+(-\sqrt[3]{2}+\sqrt[3]{3})+(-\sqrt[3]{3}+\sqrt[3]{4})$

$=\sqrt[3]{4}-\sqrt[3]{1}$ //

12. Challenge: March 2009 (Adler). Show that $n^{n-1}-1$ is divisible by $(n-1)^2$ for every positive integer "n".

$(n^{n-1}-1)=(n-1)(\underbrace{n^{n-2}+n^{n-3}+n^{n-4}+n^{n-5}+\cdots\cdots 1}_{\text{THERE ARE } (n-1) \text{ TERMS IN HERE}})$

- I'M GOING TO ADD A BUNCH OF ZERO PAIRS
- I WILL ADD (n-1) & SUBTRACT '1' FROM EACH TERM.

$=(n-1)(n^{n-2}-1+n^{n-3}-1+n^{n-4}-1+n^{n-5}-1+\cdots\cdots+n^2-1+n-1+1-1+(n-1))$

$=(n-1)((n^{n-2}-1)+(n^{n-3}-1)+(n^{n-4}-1)+\cdots\cdots(n^2-1)+(n-1)+0+(n-1)]$

$=(n-1)[(n-1)(n^{n-3}+n^{n-4}\cdots)+(n-1)(n^{n-4}+n^{n-5}+\cdots 1)+(n-1)(n^{n-6}+n^{n-5}+\cdots 1)+\cdots\cdots(n-1)(n+1)+(n-1)+(n-1)]$

$=(n-1)(n-1)[(n^{n-3}+n^{n-4}+\cdots 1)+(n^{n-4}+n^{n-5}+\cdots 1)+(n^{n-6}+n^{n-5}+\cdots 1)+\cdots\cdots(n+1)+1+1]$

13. (Challenge) Let "x" and "y" be two-digit integers such that "y" is obtained by reversing the digits of "x". The integers "x" and "y" satisfy $x^2-y^2=m^2$ for some positive integer "m". What is the value of $x+y+m$?

$x=10a+b$
$y=10b+a$

$x+y=(11a+11b)$
$x-y=(9a-9b)$

$\therefore x^2-y^2=m^2$
$11(a+b)(9)(a-b)=m^2$
$\hookrightarrow 11=(a+b)(a-b)$
$\boxed{a=6 \quad b=5}$ //

$\therefore x=56$
$y=65$
$m=9\times 11$
$=99$

$$\begin{array}{r} 56 \\ 65 \\ \hline 121 \\ 99 \\ \hline 220 \end{array}$$

$\therefore x+y+m=56+65+99$
$=220$ //