1.6 Reciprocal Of Quad. Funct.

September 18, 2018 11:06 AM

Name:______________________________ Date:____________________________

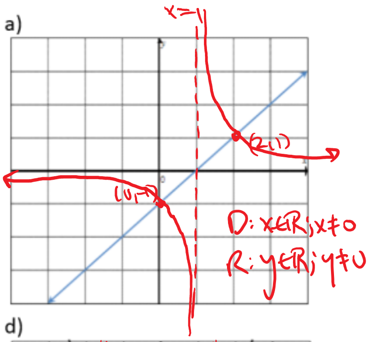
M10 Honours: Section 1.6 Reciprocals of Quadratic Functions

1. Given the equation for $y = f(x)$, find the equation for $y = \dfrac{1}{f(x)}$

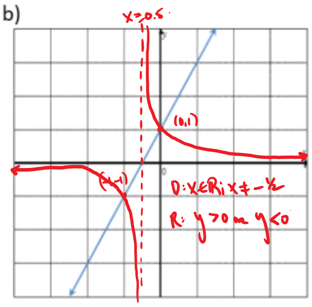
a) $y = 3x - 5$ $y = \frac{1}{3x-5}$	b) $y = \frac{2x-1}{3}$ $y = \frac{3}{2x-1}$	c) $y = \frac{3x-5}{5x-1}$ $y = \frac{5x-1}{3x-5}$	d) $y = 3x^2 + 4$ $y = \frac{1}{3x^2-4}$
e) $y = 3$ $y = \frac{1}{3}$	f) $x = -5$ N/A Not a function	g) $y = -5x^2 - 6$ $y = \frac{1}{-5x^2-6}$	h) $y = 5x^3 - 7x^2 + 22 - 6x$ $y = \frac{1}{5x^3-7x^2+22-6x}$

2. Graph the reciprocal of each function: Indicate the equations of the asymptotes, coordinate of the invariant points, domain and range of the reciprocal function.

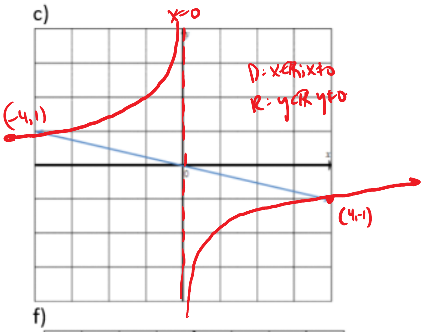
a)



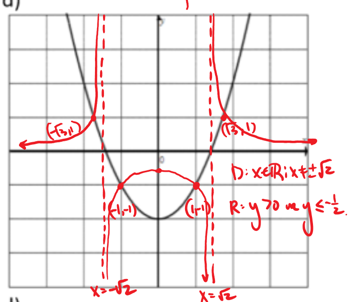
b)



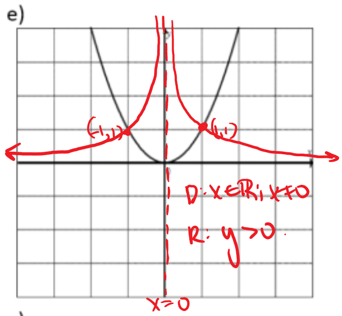
c)



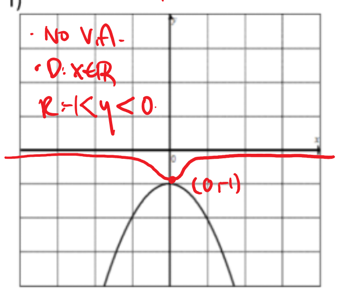
d)



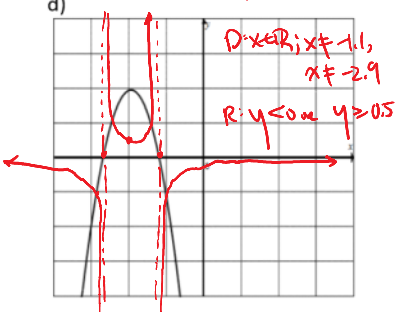
e)



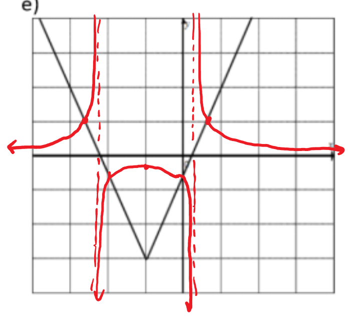
f)



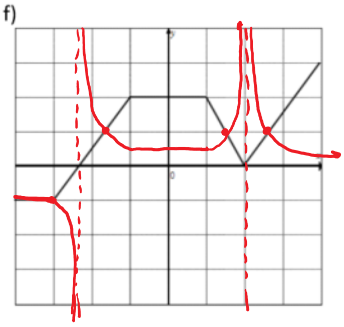
d)



e)



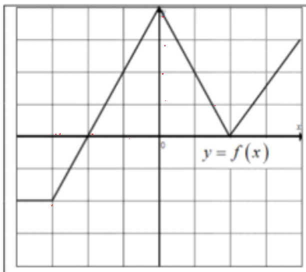
f)



3. Find the invariant points and the equation of the asymptotes:

a) $y=3x-5$ ① $1=3x-5$; $-1=3x-5$ $6=3x$; $4=3x$ $2=x$; $\frac{4}{3}=x$ $(2,1)$; $(\frac{4}{3},-1)$	b) $y=-(x-3)^2+4$ $1=-(x-3)^2+4$; $-1=-(x-3)^2+4$ $3=(x-3)^2$; $5=(x-3)^2$ $3\pm\sqrt{3}=x$; $3\pm\sqrt{5}=x$ $(3+\sqrt{3},1)\ (3-\sqrt{3},1)$; $(3+\sqrt{5},-1)(3-\sqrt{5},-1)$	c) $y=5x^2+6$ $1=5x^2+6$; $-1=5x^2+6$ $-5=5x^2$; $-7=5x^2$ No soln ; No soln. No INVARIANT PTS 6
e) $y=-3x^2-1$ $1=-3x^2-1$; $-1=-3x^2-1$ $0=-3x^2$; $-2=-3x^2$ $0=x$; $\frac{2}{3}=x^2 \rightarrow \sqrt{\frac{2}{3}}=x$ $(0,1)$; $(\sqrt{\frac{2}{3}},-1)\ (-\sqrt{\frac{2}{3}},-1)$	f) $y=-\frac{5}{2}x-11$	g) $y=-2(x-7)^2+16$

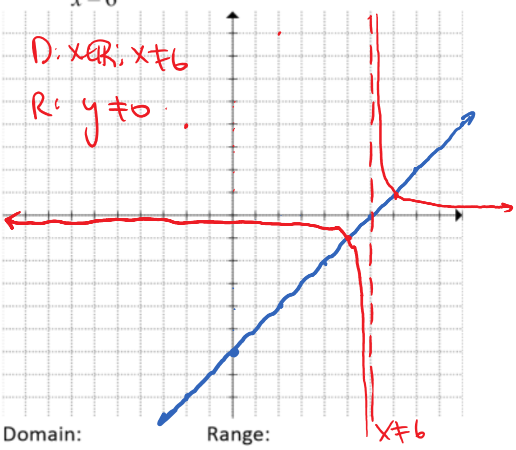
4. Given the graph of $y=f(x)$, and $g(x)=\frac{1}{f(x)}$ find the following values:



a) $g(0) = \frac{1}{4}$ $f(0)=4$	b) $g(2) =$ UNDEFINED $f(2)=0$	c) $g(1) = \frac{1}{2}$. $f(1)=2$
d) $g(k)=1 \rightarrow k=1.5, 2.5, -1.5$ $f(1.5)=f(2.5)=f(-1.5)=1$	e) $g(-3) = -0.5$ $f(-3)=-2$	f) $g(1)f(1) = 1$ $a\times\frac{1}{a}=1$
g) $g(4)\times f(4) = 1$	h) $g(4)\div f(4)$ $f(4)=3$ ∴ $\frac{1}{3}\div 3=\frac{1}{9}$ $g(4)=\frac{1}{3}$	i) $g(-2)\times f(-2)$ UNDEFINED ×0 = UNDEFINED

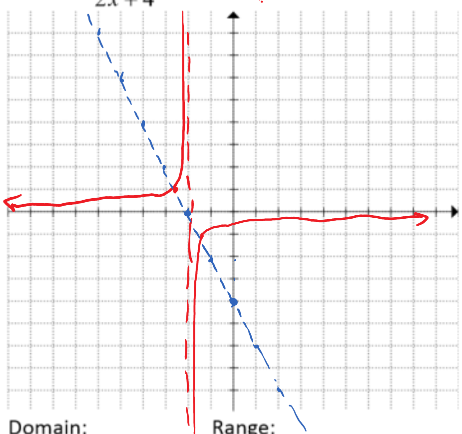
5. Graph the reciprocal functions. Indicate all asymptote, invariant points, domain and range:

a) $y=\frac{1}{x-6}$



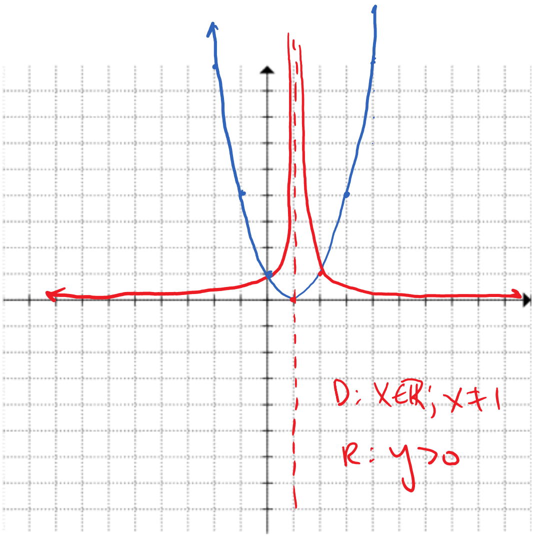
Domain: Range:

b) $y=-\frac{1}{2x+4} = \frac{1}{-2x-4}$



Domain: $x\in\mathbb{R}, x\neq -2$ Range: $y\neq 0$

c) $y = \dfrac{1}{(x-1)^2}$



D: $x \in \mathbb{R}, x \neq 1$

R: $y > 0$

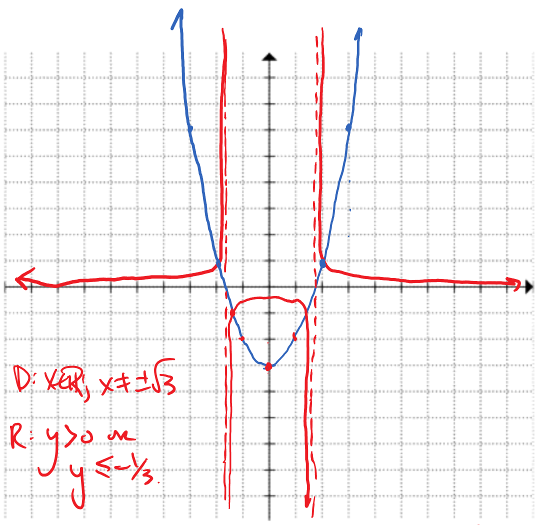
Invariant Pts: (2,1) (0,1)

Asymptotes: $x \neq 1$

Domain:

Range:

d) $y = \dfrac{1}{x^2 - 3}$



D: $x \in \mathbb{R}, x \neq \pm\sqrt{3}$

R: $y > 0$ or $y \leq -1/3$

Invariant Pts: (2,1) (-2,1) $(\sqrt{2},-1)$

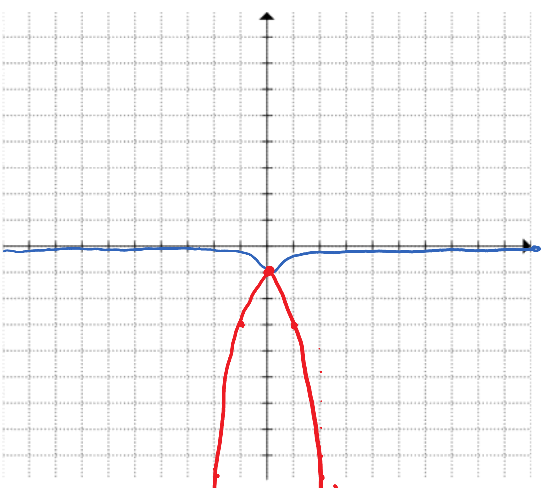
Asymptotes: $x = -\sqrt{3}$, $x = \sqrt{3}$

Domain: $(-\sqrt{2},-1)$

Range:

e) $y = -\dfrac{1}{2x^2+1}$

$y = \dfrac{1}{-2x^2-1}$



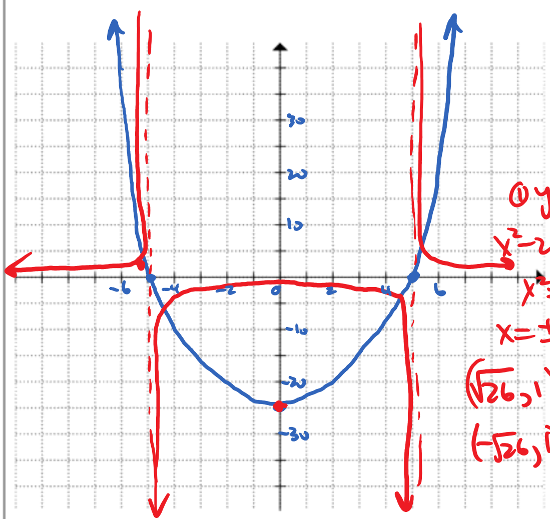
Invariant Pts: (0,-1)

Asymptotes: NONE

Domain: $x \in \mathbb{R}$

Range: $-1 \leq y < 0$

f) $y = \dfrac{1}{x^2 - 25}$



① $y = 1$

$x^2 - 25 = 1$

$x^2 = 26$

$x = \pm\sqrt{26}$

$(\sqrt{26}, 1)$

$(-\sqrt{26}, 1)$

$y = -1$

$x^2 - 25 = -1$

$x^2 = 24$

$x = \pm\sqrt{24}$

$x = \pm 2\sqrt{6}$

$(2\sqrt{6}, -1)$

$(-2\sqrt{6}, -1)$

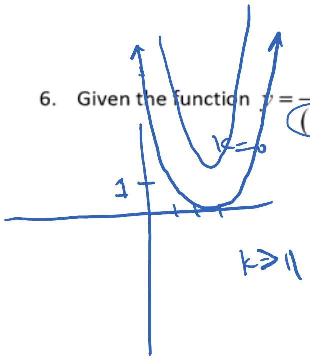
Invariant Pts:

Asymptotes: $x = \pm 5$

Domain: $x \in \mathbb{R}; x \neq \pm 5$

Range: $y > 0$ or $y \leq -\frac{1}{25}$

6. Given the function $y = \dfrac{1}{(x-3)^2 + k}$, for what values of "k" will the graph not have any invariant points?



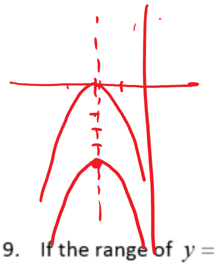
$k > 1$

7. Given the function $y = \dfrac{-1}{(x-2)^2 + k}$, for what values of "k" will the graph have four invariant points?

8. Given the function $y = \dfrac{-1}{(x+2)^2 + 4 - k}$, for what values of "k" will the graph not have any vertical aymptotes?

$y = \dfrac{1}{-(x+2)^2 - 4 + k}$

$-(x+2)^2 - 4 + 8$



② Bottom is a Q.E.
No x-int → recip will have no VA

$b^2 - 4ac < 0$

$-(x+2)^2 - 4 + k$

$-(x^2 + 4x + 4) - 4 + k$

$-x^2 - 4x - 8 + k$

$a = -1 \quad b = -4 \quad c = -8 + k$

$(-4)^2 - 4(-1)(-8+k) < 0$

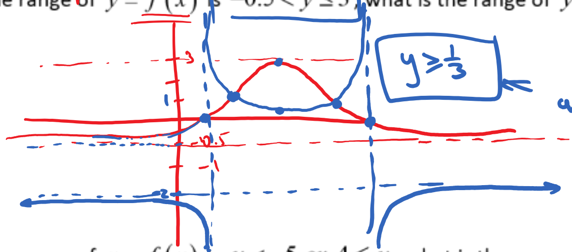
$16 + 4(-8+k) < 0$

$16 - 32 + 4k < 0$

$4k < 16$

$\boxed{k < 4}$

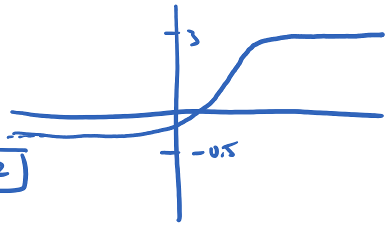
9. If the range of $y = f(x)$ is $\boxed{-0.5 < y \le 3}$, what is the range of $y = \dfrac{1}{f(x)}$? Justify your answer



$\boxed{y \ge \frac{1}{3}}$

or

$\boxed{y < -2}$



10. If the range of $y = f(x)$ is $y < -5$ or $4 \le y$, what is the range of $y = \dfrac{1}{f(x)}$? Justify your answer