Question:

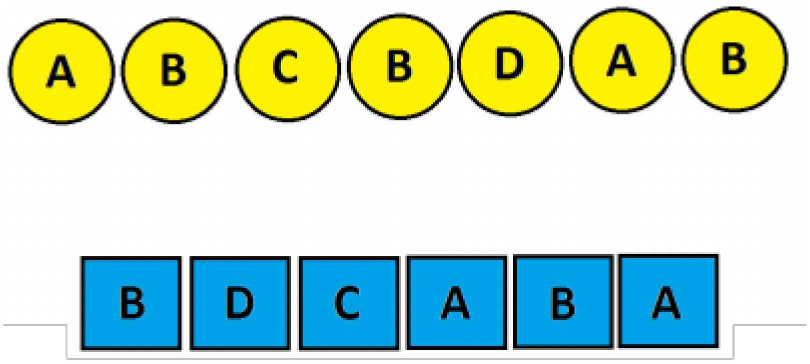
Bob the Beaver plays a game against Sam the Squirrel.
At first Bob removes four numbers from the list 1, 2, 3, 4, 5, 6, 7, 8.
Then Sam deletes two of the remaining numbers.
Bob wants the positive difference between the remaining two numbers to be as large as possible. Sam wants this positive difference to be as small as possible. Both Bob and Sam know what the other player is trying to acheive.
What is the positive difference between the last two numbers if both Bob and Sam play as well as possible?

Possible Answers:

1
2
3
4

Question:

You can connect a circle and a square that both have the same letter in them using a *straight* line.



What is the maximum number of such connections you can make without crossing any lines?

Possible Answers:

2
3
4
5

Question #3) (2013)

When three students, Al, Betty, and Charlie, compete in a competition, there are 13 possible orders of finish, allowing for the possibility of ties. These possibilities are illustrated in the chart below:

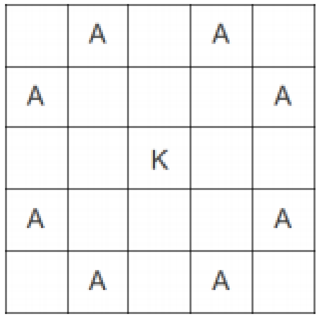
First	A	A	B	B	C	C	ABC	AB	AC	BC	A	B	C
Second	B	C	A	C	A	B		C	B	A	BC	AC	AB
Third	C	B	C	A	B	A							

When four students, David, Erin, Frank, and Greg, compete in a competition, how many possible orders of finish are there, allowing for the possibility of ties?

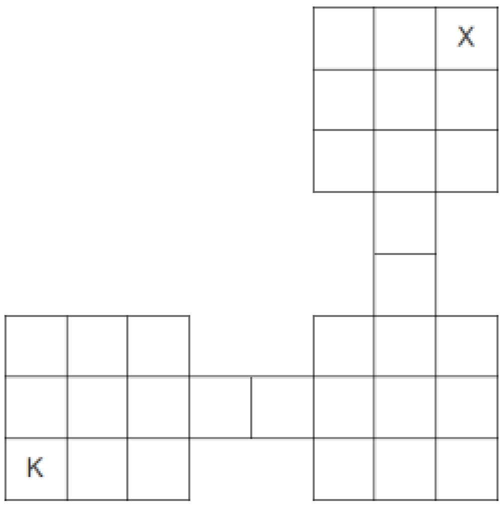
Question #4) (2012

Question:

There are eight ways a knight can jump. If it starts at the position marked K on the grid below, in one move it can jump to any one of the eight positions marked A.



Suppose you have three 3-by-3 grids connected in the following way by two bridges of length two.



What is the fewest number of moves needed to move the knight from position K to position X without ever leaving the grids or bridges?

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Question 5)

For each positive integer N, an *Eden sequence* from $\{1, 2, 3, \ldots, N\}$ is defined to be a sequence that satisfies the following conditions:

(i) each of its terms is an element of the set of consecutive integers $\{1, 2, 3, \ldots, N\}$,

(ii) the sequence is increasing, and

(iii) the terms in odd numbered positions are odd and the terms in even numbered positions are even.

For example, the four Eden sequences from $\{1, 2, 3\}$ are

$$1 \qquad 3 \qquad 1, 2 \qquad 1, 2, 3$$

(a) Determine the number of Eden sequences from $\{1, 2, 3, 4, 5\}$.

(b) For each positive integer N, define $e(N)$ to be the number of Eden sequences from $\{1, 2, 3, \ldots, N\}$. If $e(17) = 4180$ and $e(20) = 17710$, determine $e(18)$ and $e(19)$.

Question #6

A *multiplicative partition* of a positive integer $n \geq 2$ is a way of writing n as a product of one or more integers, each greater than 1. Note that we consider a positive integer to be a multiplicative partition of itself. Also, the order of the factors in a partition does not matter; for example, $2 \times 3 \times 5$ and $2 \times 5 \times 3$ are considered to be the same partition of 30. For each positive integer $n \geq 2$, define $P(n)$ to be the number of multiplicative partitions of n. We also define $P(1) = 1$. Note that $P(40) = 7$, since the multiplicative partitions of 40 are 40, 2×20, 4×10, 5×8, $2 \times 2 \times 10$, $2 \times 4 \times 5$, and $2 \times 2 \times 2 \times 5$.

(a) Determine the value of $P(64)$.

(b) Determine the value of $P(1000)$.

(c) Determine, with proof, a sequence of integers $a_0, a_1, a_2, a_3, \ldots$ with the property that

$$P(4 \times 5^m) = a_0 P(2^m) + a_1 P(2^{m-1}) + a_2 P(2^{m-2}) + \cdots + a_{m-1} P(2^1) + a_m P(2^0)$$

for every positive integer m.

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Question #7) (2014)

The *positive difference list (PDL)* of a set of integers is a list, written in increasing order, of the positive differences between all possible pairs of integers in the set. For example, the set $\{2, 5, 12\}$ produces a PDL consisting of three integers $3, 7, 10$ which are distinct, while the set $\{3, 4, 6, 9\}$ produces a PDL consisting of six integers $1, 2, 3, 3, 5, 6$ which are not distinct.

(a) What is the PDL of the set of integers $\{3, 6, 13, 21, 32\}$?

(b) Suppose that $x > 16$ and the sum of the integers in the PDL of the set $\{1, 4, 9, 16, x\}$ is 112. Determine the value of x.

(c) State a set of integers of the form $\{3, q, r, s, 14\}$ with $3 < q < r < s < 14$ for which the PDL contains no repeated integers.

(d) Prove that the PDL of every set of integers of the form $\{3, q, r, s, t\}$ with $3 < q < r < s < t$ and $t < 14$ contains repeated integers.

Question 8)

Fiona plays a game with jelly beans on the number line. Initially, she has N jelly beans, all at position 0. On each turn, she must choose one of the following moves:

- Type 1: She removes two jelly beans from position 0, eats one, and puts the other at position 1.
- Type i, where i is an integer with $i \geq 2$: She removes one jelly bean from position $i-2$ and one jelly bean from position $i-1$, eats one, and puts the other at position i.

The positions of the jelly beans when no more moves are possible is called the *final state*. Once a final state is reached, Fiona is said to have won the game if there are at most three jelly beans remaining, each at a distinct position and no two at consecutive integer positions. For example, if $N = 7$, Fiona wins the game with the sequence of moves

Type 1, Type 1, Type 2, Type 1, Type 3

which leaves jelly beans at positions 1 and 3. A different sequence of moves starting with $N = 7$ might not win the game.

(a) Determine an integer N for which it is possible to win the game with one jelly bean left at position 5 and no jelly beans left at any other position.

(b) Suppose that Fiona starts the game with a fixed unknown positive integer N. Prove that if Fiona can win the game, then there is only one possible final state.

(c) Determine, with justification, the closest positive integer N to 2014 for which Fiona can win the game.