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Math 12 Enriched: HW Section ~~1.5~~ 2.5 Combined Transformations

1. Indicate what the function $y = f(x)$ will become after each transformation in the specified order:

a) 1. Horizontal Shift of 3 units left $x \to x+3$ 2. Horizontal expansion and reflection by a factor 3 $x \to -\frac{1}{3}x$ $y = f(x+3)$ $\boxed{y = f(-\frac{1}{3}x+3)}$	b. 1. Horizontal Expansion and reflection by a factor of 3 $x \to -\frac{1}{3}x$ 2. Horizontal Shift of 3 lefts left $x \to x+3$ $y = f(-\frac{1}{3}x)$ $y = f(-\frac{1}{3}(x+3)) \Rightarrow \boxed{y = f(-\frac{1}{3}x-1)}$
c) A vertical compression by a factor of -0.75 Vertical shift of 8 units up $y \to -\frac{4}{3}y$ $y \to y-8$ $-\frac{4}{3}y = f(x)$ $-\frac{4}{3}(y-8) = f(x) \Rightarrow \boxed{y = -\frac{3}{4}f(x)+8}$	d) Vertical shift of 8 units up and then a vertical compression by a factor of -0.75 $y \to y-8$ $y \to -\frac{4}{3}y$ $y = f(x)+8$ $\boxed{y = -\frac{3}{4}f(x)-6}$
e) A vertical expansion by a factor of 2 and then a reflection over the x-axis. Then a horizontal compression by a factor of 0.25. $y \to \frac{1}{2}y$ $y = 2f(x)$ $y \to -y$ $y = -2f(x)$ $x \to 4x$ $\boxed{y = -2f(4x)}$	f) A horizontal shift of 3 units left and 2 units up. Then a reflection on both axis. Then a HE of 3 and VC of 0.3. $x \to x+3 \to y = f(x+3)$ $y \to y-2 \to y = f(x+3)+2$ $x \to -x \to y = f(-x+3)+2$ $y \to -y \to y = -f(-x+3)-2$ $x \to \frac{1}{3}x \to y = -f(-\frac{x}{3}+3)-2$ $y \to \frac{10}{3}y \to \boxed{y = -\frac{3}{10}f(-\frac{1}{3}x+3)-\frac{3}{5}}$

2. When two transformations are performed in different orders, will the resulting function always be the same or always different? Explain:

It depends. It can sometimes be the same & sometimes different. For example, whether I perform a HS. or V.S. first does not matter, but whether I perform a H.C. or H.S. first matters!!!
1a. and 1b. is a good example of this

3. The function $y = \sqrt{x}$ is horizontally expanded by a factor of 4. With what VE/VC will result in the same equation?

H.E. by 4 $x \to \frac{1}{4}x$ $y = \sqrt{\frac{1}{4}x} = \frac{1}{2}\sqrt{x}$ OR

$\boxed{\text{V.C. by } \frac{1}{2}}$ $y \to 2y$ $y = \frac{1}{2}\sqrt{x}$

4. For what factor "K" will the transformation of $y = f(x) \to ky = f(x)$ transform the function from

$y = x^2 \to y = (4x)^2$?

$y = x^2 \to y = 16(x)^2$

$\Rightarrow \frac{y}{16} = x^2$ $\boxed{k = \frac{1}{16}}$

5. What is the transformation required to convert $y = (x-3)^2 \to y = (4x-12)^2$? Name two different sets of solutions:

$y = (x-3)^2 \to y = 16(x-3)^2$

Approach 1
$\boxed{\begin{array}{l}\text{H.S. 9R} \\ \text{H.C. by } \frac{1}{4}\end{array}}$ $x \to x-9$ $y = (x-12)^2$; $x \to 4x$ $y = (4x-12)^2$

Approach 2
$\boxed{\text{V.E. by 16}}$ $y \to 16y$ $y = 16(x-3)^2 = (4x-12)^2$

6. Indicate all the transformations that ~~is~~ are required to change from $y=f(x)$ to the equation give:

a) $y=2f(3x-1)+1$

H.S. 1R	$x\to x-1$	$y=f(x-1)$	$(a+1,b)$
H.C. by $\frac{1}{3}$	$x\to 3x$	$y=f(3x-1)$	$(\frac{a+1}{3},b)$
V.C. by 2	$y\to \frac{1}{2}y$	$y=2f(3x-1)$	$(\frac{a+1}{3},2b)$
V.S. 1U	$y\to y-1$	$y=2f(3x-1)+1$	$(\frac{a+1}{3},2b+1)$

b) $y=-\frac{2}{3}f(3x+12)+1$

H.S. 12L	$x\to x+12$	$y=f(x+12)$
H.C. by $\frac{1}{3}$	$x\to 3x$	$y=f(3x+12)$
V.R.	$y\to -y$	$y=-f(3x+12)$
V.C. by $\frac{2}{3}$	$y\to \frac{3}{2}y$	$y=-\frac{2}{3}f(3x+12)$
V.S. 1U	$y\to y-1$	$y=-\frac{2}{3}f(3x+12)+1$

c) $y=12-\frac{3}{5}f(8-2x)$

H.S. 8L	$x\to x+8$	$y=f(x+8)$
H.R. and H.C. by $\frac{1}{2}$	$x\to -2x$	$y=f(-2x+8)$
V.R. and V.C. by $\frac{3}{5}$	$y\to -\frac{5}{3}y$	$y=-\frac{3}{5}f(-2x+8)$
V.S. 12U	$y\to y+12$	$y=-\frac{3}{5}f(-2x+8)+12$

d) $\frac{1}{2}x=f(4y+1)$

Inverse reflection	$y=f(x)\to x=f(y)$	$x=f(y)$
V.S. 1D	$y\to y+1$	$x=f(y+1)$
V.C. by $\frac{1}{4}$	$y\to 4y$	$x=f(4y+1)$
H.E. by 2	$x\to \frac{1}{2}x$	$\frac{1}{2}x=f(4y+1)$

e) $-0.\bar{2}y=f(3x-4)+1$

$-\frac{2}{9}y=f(3x-4)+1$

H.S. 4R	$x\to x-4$	$y=f(x-4)$
H.C. by $\frac{1}{3}$	$x\to 3x$	$y=f(3x-4)$
V.R. and V.	$y\to -\frac{9}{2}y$	$-\frac{9}{2}y=f(3x-4)$
V.S. 1U	$y\to y-1$	$-\frac{9}{2}y=f(3x-4)+$

f) $0.\bar{3}(x-1)=f\left(\frac{y}{2}+3\right)$

$\frac{1}{3}(x-1)=f(\frac{y}{2}+3)$

Inverse function	$y=f(x)\to x=f(y)$	$x=f(y)$
V.S. 3D	$y\to y+3$	$x=f(y+3)$
V.C. by 2	$y\to \frac{1}{2}y$	$x=f(\frac{y}{2}+3)$
H.E. by 3	$x\to \frac{1}{3}x$	$\frac{1}{3}x=f(\frac{y}{2}+3)$
H.S. 1U	$x\to x-1$	$\frac{1}{3}(x-1)=f(\frac{y}{2}+3)$

7. Given the four transformations in the given order, what will function $y=f(x)$ result in?

H.R. and H.E. by -2   V.E. by -4   H.S. 4L   V.S. 12U

a) 1st) $x\to -\frac{1}{2}x$ 2nd) $y\to \frac{-y}{4}$ 3rd) $x\to x+4$ 4th) $y\to y-12$

$y=f(-\frac{1}{2}x)$ $y=-4f(-\frac{1}{2}x)$ $y=-4f(-\frac{1}{2}x-2)$ $\boxed{y=-4f(-\frac{1}{2}x-2)+12}$

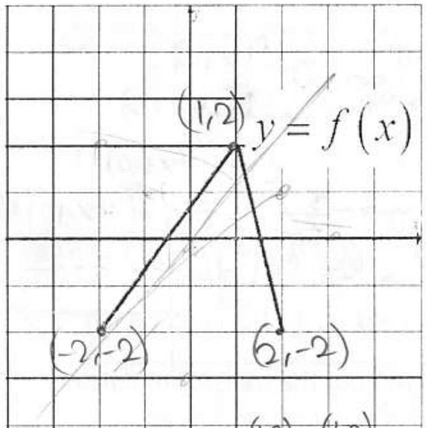
H.S. 2L   H.R   V.S 5D   V.E. by -2   Inverse reflection   V.S. 4D

b) 1st) $x\to 2-x$ 2nd) $y\to 5-\frac{1}{2}y$ 3rd) $y\leftrightarrow x$ 4th) $y\to y+4$

$y=f(2-x)$ $5-\frac{1}{2}y=f(2-x)$ $5-\frac{1}{2}x=f(2-y)$ $\boxed{5-\frac{1}{2}x=f(-y-2)}$

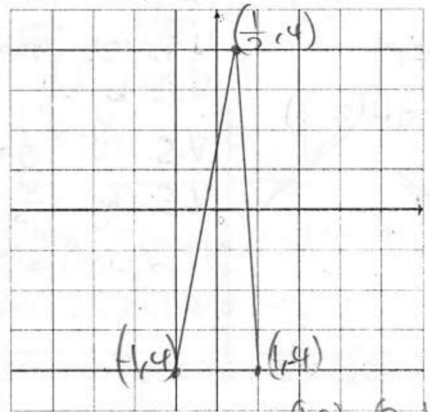
Don't forget to lable new graph!

8. Given the graph of $y = f(x)$, draw the graph of the following functions:



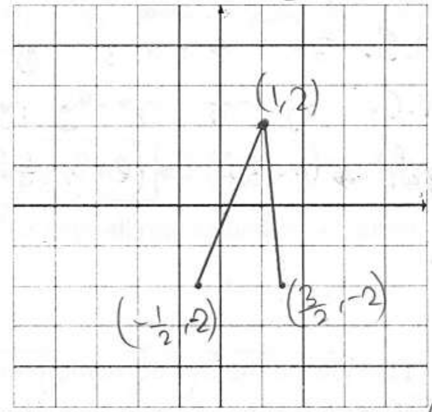
H.C. by $\frac{1}{2}$
V.C. by 2
$(1,2) \to (\frac{1}{2}, 4)$
$(-2,-2) \to (-1,-4)$
$(2,-2) \to (1,-4)$

a) $y = 2f(2x)$



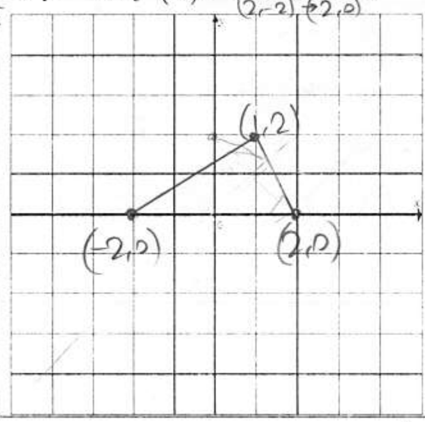
H.S. 1R
H.C by $\frac{1}{2}$
$(1,2) \to (1,2)$
$(-2,-2) \to (-\frac{1}{2},-2)$
$(2,-2) \to (\frac{3}{2},-2)$

b) $y = f(2x-1)$



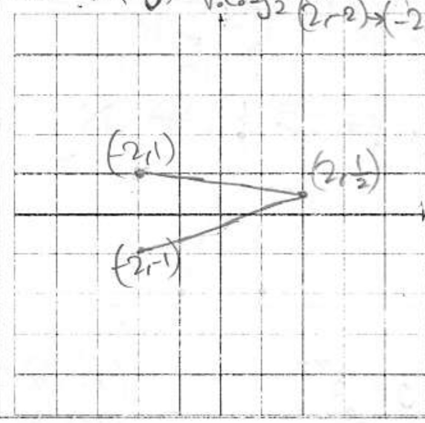
V.C. by $\frac{1}{2}$
V.S. 1U
$(1,2) \to (1,2)$
$(-2,-2) \to (-2,0)$
$(2,-2) \to (2,0)$

c) $y = 0.5f(x)+1$



Inverse
V.C. by $\frac{1}{2}$
$(1,2) \to (2,\frac{1}{2})$
$(-2,-2) \to (-2,-1)$
$(2,-2) \to (-2,1)$

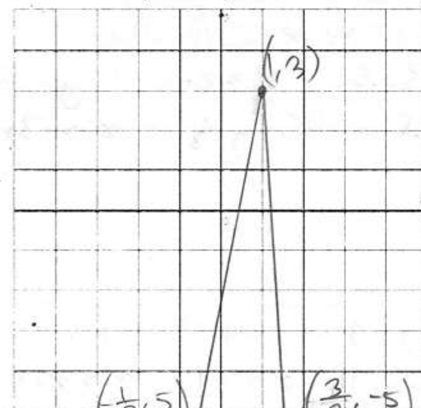
d) $x = f(2y)$



f) $y = 2f(2x-1)-1$

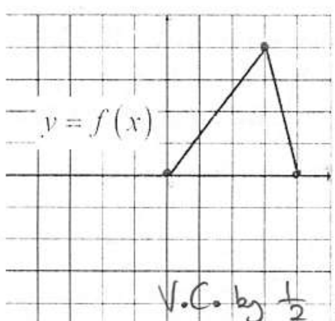
$(1,2) \to (1,3)$
$(-2,-2) \to (-\frac{1}{2},-5)$
$(2,-2) \to (\frac{3}{2},-5)$

H.S. 1R
H.C. by $\frac{1}{2}$
V.C. by 2
V.S. 1D



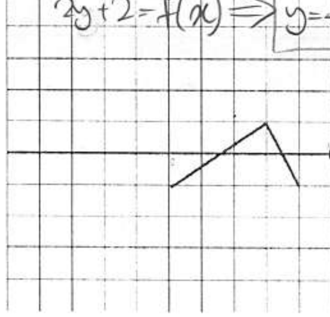
9. Given the graph of $y = f(x)$ on top, what is the equation of the corresponding graph below it:

a)

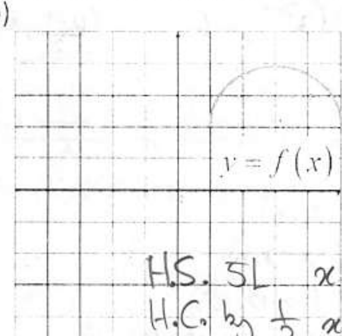


V.C. by $\frac{1}{2}$ $y \to 2y$
V.S. 1D $y \to y+1$

$2y+2 = f(x) \Rightarrow \boxed{y = \frac{1}{2}f(x)-1}$

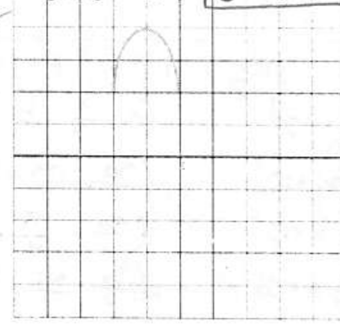


b)

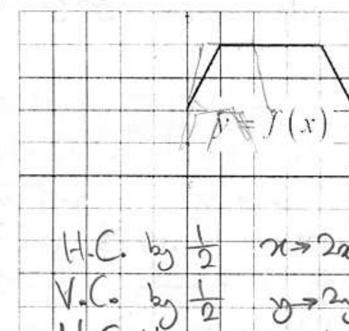


H.S. 5L $x \to x+5$
H.C. by $\frac{1}{2}$ $x \to 2x$

$y = f(x+5) \Rightarrow \boxed{y = f(2x+5)}$

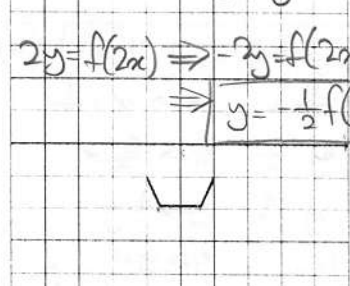


c)



H.C. by $\frac{1}{2}$ $x \to 2x$
V.C. by $\frac{1}{2}$ $y \to 2y$
H.S. 1L $x \to x+1$
V.R. $y \to -y$

$2y = f(2x) \Rightarrow -2y = f(2x+2)$
$\Rightarrow \boxed{y = -\frac{1}{2}f(2x+2)}$



10. Point (e,f) is on the graph of $y = f(x)$, what point must be on the following functions:

a) $y = -\frac{1}{4} f(x-3)$ H.S. 3R $\quad x \to x-3 \quad y = f(x-3)$ V.C. by $-\frac{1}{4}$ $\quad y \to -4y \quad y = -\frac{1}{4} f(x-3)$ $(e,f) \to (e+3, f) \to \boxed{(e+3, -\frac{1}{4}f)}$	b) $\frac{-3}{4} y = f(10-4x)+1$ H.S. 10L $\quad x \to x+10 \quad y = f(x+10)$ H.C. by $-\frac{1}{4}$ $\quad x \to -4x \quad y = f(-4x+10)$ V.S. 1U $\quad y \to y-1 \quad y = f(-4x+10)+1$ V.E. by $-\frac{4}{3}$ $\quad y \to -\frac{3}{4}y \quad -\frac{3}{4}y = f(-4x+10)+1$ $(e,f) \to (e-10, f) \to (\frac{10-e}{4}, f) \to (\frac{10-e}{4}, f+1) \to \boxed{(\frac{10-e}{4}, -\frac{4}{3}f - \frac{4}{3})}$

11. Indicate the transformation required to go from the left function to the right. List the transformation in order:

a) $y = \sqrt{x} \to y = \sqrt{5-3x}$

①	H.S. 5L	$x \to x+5$	$y = \sqrt{x+5}$	$(a-5, b)$
②	H.R. and H.C. by $\frac{1}{3}$	$x \to -3x$	$y = \sqrt{-3x+5}$	$(\frac{5-a}{3}, b)$

b) $y = 3^x \to y = 4(3^{2x+1}) - 6$

①	H.S. 1L	$x \to x+1$	$y = 3^{x+1}$	$(a-1, b)$
②	H.C. by $\frac{1}{2}$	$x \to 2x$	$y = 3^{2x+1}$	$(\frac{a-1}{2}, b)$
③	V.E by 4	$y \to \frac{1}{4}y$	$y = 4(3^{2x+1})$	$(\frac{a-1}{2}, 4b)$
④	V.S. 6D	$y \to y+6$	$y = 4(3^{2x+1}) - 6$	$(\frac{a-1}{2}, 4b-6)$

c) $y = \sqrt{x} \to y = 12\sqrt{-x-12} + 11$

①	H.R.	$x \to -x$	$y = \sqrt{-x}$	$(-a, b)$
②	H.S. 12L	$x \to x+12$	$y = \sqrt{-x-12}$	$(-a-12, b)$
③	V.E. by 12	$y \to \frac{1}{12}y$	$y = 12\sqrt{-x-12}$	$(-a-12, 12b)$
④	V.S. 11U	$y \to y-11$	$y = 12\sqrt{-x-12} + 11$	$(-a-12, 12b+11)$

d) $y = \lvert 2x+1 \rvert \to y = 3\lvert \frac{4}{5}x + 12 \rvert - 1$

①	H.S. $\frac{11}{2}$ L	$x \to x + \frac{11}{2}$	$y = \lvert 2x+12 \rvert$	$(a - \frac{11}{2}, b)$
②	H.E. by $\frac{5}{2}$	$x \to \frac{2}{5}x$	$y = \lvert \frac{4}{5}x + 12 \rvert$	$(\frac{5a}{2} - \frac{55}{4}, b)$
③	V.E. by 3	$y \to \frac{1}{3}y$	$y = 3\lvert \frac{4}{5}x + 12 \rvert$	$(\frac{5a}{2} - \frac{55}{4}, 3b)$
④	V.S. 1D	$y \to y+1$	$y = 3\lvert \frac{4}{5}x + 12 \rvert - 1$	$(\frac{5a}{2} - \frac{55}{4}, 3b-1)$

12. The domain and range of $y=f(x)$ is $D:\{x\ge 4\}$ & $R:\{y\ge 0\}$. What is the domain and range for $y=f(x+5)$?

Range does not change since we are performing a horizontal shift only.

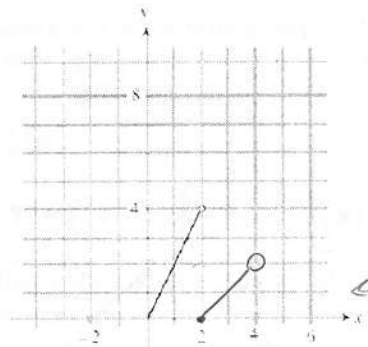
$y=f(x) \to y=f(x+5)$

H.S. 5L $\quad x\to x+5$

Since graph is shifted 5 left, our domain goes 5 left as well.

$$\boxed{\begin{aligned} D&: x\ge -1 \\ R&: y\ge 0 \end{aligned}}$$

13. Part of the graph for $y=f(x)$ is shown, $0\le x<2$. If $g(x+2)=\dfrac{1}{2}f(x)$ for all real values of "x", draw the graph of $g(x)$ for the intervals $-2\le x<0$ and $2\le x<6$.



$g(x+2)=\frac{1}{2}f(x)$

$\boxed{\begin{array}{l}\text{V.C. by } \frac{1}{2} \\ \text{H.S. 2R}\end{array}}$ $\quad x\to x-2 \qquad g(x+2)\to g(x)=\frac{1}{2}f(x-2)$

14. Challenge: if $x=\dfrac{1}{2}$ then the value of the product: $(1+x)(1+x^2)(1+x^4)\times\ldots\times\cancel{(1+x^{2^{n-1}})}\times\ldots\times(1+x^{128})$ is $2-2^k$. What is the value of "k"? CNML 1994 4 6

$n=1 \quad n=2 \quad n=3 \quad \ldots\times\left(1+x^{(2^{n-1})}\right)\times\ldots \quad n=n \quad n=8$

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{2^4}\right)\left(1+\frac{1}{2^8}\right)\left(1+\frac{1}{2^{16}}\right)\left(1+\frac{1}{2^{32}}\right)\left(1+\frac{1}{2^{64}}\right)\left(1+\frac{1}{2^{128}}\right)=2-2^k$$

LOOK FOR HINTS! DIFFERENCE OF SQUARES!

multiply both sides by $\left(1-\frac{1}{2}\right)$

$(a-b)(a+b)=a^2-b^2$

$$\underbrace{\left(1-\frac{1}{2}\right)\left(1+\frac{1}{2}\right)}\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{2^4}\right)\cdots\left(1+\frac{1}{2^{128}}\right)=\left(2-2^k\right)\left(1-\frac{1}{2}\right)$$

$$\underbrace{\left(1-\frac{1}{2^2}\right)\left(1+\frac{1}{2^2}\right)}\left(1+\frac{1}{2^4}\right)\cdots\left(1+\frac{1}{2^{128}}\right)=\left(2-2^k\right)\left(\frac{1}{2}\right)$$

$$\left(1-\frac{1}{2^4}\right)\left(1+\frac{1}{2^4}\right)\left(1+\frac{1}{2^8}\right)\cdots\left(1+\frac{1}{2^{128}}\right)=\frac{1}{2}\left(2-2^k\right)$$

$$\vdots$$

$$1-\frac{1}{2^{256}}=\frac{1}{2}\left(2-2^k\right)$$

$$\frac{1}{2^{256}}=2^{k-1} \implies k-1=-256 \implies \boxed{k=-255}$$