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Math 12 Honours: Section 2.4 Expansions and Compressions of Functions

1. Indicate the transformation from the function on the left to the function on the right. What are all the different possible transformations:

a) $y=|x| \to y=3|2x|$

H.C. by $\frac{1}{2}$ $x\to 2x$ $y=|2x|$ $(\frac{1}{2}a, b)$
V.E by 3 $y\to \frac{1}{3}y$ $y=3|2x|$ $(\frac{1}{2}a, 3b)$

OR

V.E. by 3 $(a, 3b)$
H.C. by $\frac{1}{2}$ $(\frac{1}{2}a, 3b)$

b) $y=\sqrt{x} \to y=\sqrt{4x}$

H.C. by $\frac{1}{4}$ $x\to 4x$ $y=\sqrt{4x}$ $(\frac{1}{4}a, b)$ ← $b=\frac{\sqrt{a}}{2}$

OR

V.E by 2 $y\to \frac{1}{2}y$ $y=2\sqrt{x}=\sqrt{4x}$ $(a, 2b)$ ← $b=\frac{\sqrt{a}}{2}$

c) $y=\frac{1}{2x-3} \to y=\frac{3}{4x-3}$

H.C. by $\frac{1}{2}$ $x\to 2x$ $y=\frac{1}{4x-3}$
V.E by 3 $y\to \frac{1}{3}y$ $y=\frac{3}{4x-3}$

OR

other way around

d) $y=x^2 \to y=4x^2-12x+9$

$y=(2x-3)^2$

H.S. 3R $x\to x-3$ $y=(x-3)^2$
H.C. by $\frac{1}{2}$ $x\to 2x$ $y=(2x-3)^2$

} better & faster

Remember! always circle x & change $2x\to 2(x-3)$!!

OR

H.C. by $\frac{1}{2}$ $x\to 2x$ $y=(2x)^2$
H.S. by $\frac{3}{2}$R $x\to x-\frac{3}{2}$ $y=\left(2(x-\frac{3}{2})\right)^2 \Rightarrow y=(2x-3)^2$

e) $y=x^3 \to y=8x^3-12x^2+6x-1$

$y=(2x-1)^3=8x^3-12x^2+6x-1$

H.S. 1R $x\to x-1$ $y=(x-1)^3$
H.C. by $\frac{1}{2}$ $x\to 2x$ $y=(2x-1)^3$

OR

H.C. by $\frac{1}{2}$
H.S. $\frac{1}{2}$R

f) $y=2^{x+4} \to y=4(2^x)^3-1 \Rightarrow y=4(2^{3x})-1 \Rightarrow y=2^{3x+2}-1$

H.S. 4R $x\to x-4$ $y=2^x$
H.C. by $\frac{1}{3}$ $x\to 3x$ $y=2^{3x}$
V.E by 4 $y\to \frac{1}{4}y$ $y=4(2^{3x})$
V.S. 1D $y\to y+1$ $y=4(2^{3x})-1$

OR

H.S. 2R $x\to x-2$ $y=2^{x+2}$
H.C. by $\frac{1}{3}$ $x\to 3x$ $y=2^{3x+2}$
V.S. 1D $y\to y+1$ $y=2^{3x+2}-1$

Note that each approach can also be done in two ways in which a H.C. is done before the first H.S.

2. Indicate the transformation for each of the following:

a) $y=f(x)\to y=2f(x+1)$ H.S. 1L $x\to x+1$ $y=f(x+1)$ V.C. by factor of 2 $y\to \frac{1}{2}y$ $y=2f(x+1)$	b) $y=f(x)\to y=f(2x)+5$ H.C. by $\frac{1}{2}$ $x\to 2x$ $y=f(2x)$ V.S. 5U $y\to y-5$ $y=f(2x)+5$
c) $y=f(x)\to y=\frac{1}{3}f(2x)-4$ H.C. by $\frac{1}{2}$ $x\to 2x$ $y=f(2x)$ V.C. by $\frac{1}{3}$ $y\to 3y$ $3y=f(2x)$ V.S. 4D $y\to y+4$ $3(y+4)=f(2x)$ $3y+12=f(2x)$ $y=\frac{1}{3}f(2x)-4$	d) $y=f(x)\to y=\frac{f(2x-4)}{4} \Rightarrow y=\frac{1}{4}f(2x-4)$ H.S. 4R $x\to x-4$ $y=f(x-4)$ H.C. by $\frac{1}{2}$ $x\to 2x$ $y=f(2x-4)$ V.C. by $\frac{1}{4}$ $y\to 4y$ $y=\frac{1}{4}f(2x-4)$
e) $y=f(x)\to y=\frac{3}{2}f\left(\frac{2}{3}x-3\right)$	f) $y=f(x)\to y=1-\frac{4}{3}f\left(\frac{2x+1}{3}\right)$

H.S. 3R $x\to x-3$ $y=f(x-3)$ H.E by $\frac{3}{2}$ $x\to\frac{2}{3}x$ $y=f(\frac{2}{3}x-3)$ V.E. by $\frac{3}{2}$ $y\to\frac{2}{3}y$ $y=\frac{3}{2}f(\frac{2}{3}x-3)$	H.E. by $\frac{3}{2}$ $x\to\frac{2}{3}x$ $y=f(\frac{2}{3}x)$ H.S. by $\frac{1}{2}$ L $x\to x+\frac{1}{2}$ $y=f(\frac{2x+1}{3})$ V.E. by $\frac{-4}{3}$ $y\to\frac{-3}{4}y$ $y=\frac{-4}{3}f(\frac{2x+1}{3})$ V.S. 1U $y\to y-1$ $y=1-\frac{4}{3}f(\frac{2x+1}{3})$
g) $y=f(x)\to y=3-5f(0.5x-4)$ H.S. 4R $x\to x-4$ $y=f(x-4)$ H.E by 2 $x\to\frac{1}{2}x$ $y=f(\frac{1}{2}x-4)$ V.E. by 5 $y\to\frac{1}{5}y$ $y=5f(\frac{1}{2}x-4)$ V.R. $y\to -y$ $y=-5f(\frac{1}{2}x-4)$ V.S. 3U $y\to y-3$ $y=3-5f(\frac{1}{2}x-4)$	h) $y=f(x)\to y=\dfrac{-5+2f(0.3x-2)}{3}$ H.S. 2R $x\to x-2$ $y=f(x-2)$ H.E by $\frac{10}{3}$ $x\to 0.3x$ $y=f(0.3x-2)$ V.C. by $\frac{2}{3}$ $y\to\frac{3}{2}y$ $y=\frac{2}{3}f(0.3-2)$ V.S. $\frac{5}{3}$ D $y\to y+\frac{5}{3}$ $y=\dfrac{2f(0.3-2)-5}{3}$

3. Given $y=f(x)$, indicate the new equation after each transformation in the order stated:

a) $f(x)=2x+3$ $y=2(\frac{1}{3}x)+3$ $\boxed{y=\frac{2}{3}x+8}$	1. A horizontal expansion by a factor of 3 $x\to\frac{1}{3}x$ 2. Then shifted up by 5 units $y\to y-5$
b) $f(x)=(x-3)^2-4$ $-2y=(x-3)^2-4$ $-2y=(x-1)^2-4$ $-2y-12=(x-1)^2-4$ $\Rightarrow -2y-12=x^2-2x-3$ $-2y=x^2-2x+9$ $\boxed{y=-\frac{1}{2}x^2+x-\frac{9}{2}}$	1. A vertical reflection and compression by a factor of 0.5 $y\to -2y$ 2. A shift of 2 units left, and $x\to x+2$ 3. Shift of 6 units down $y\to y+6$
c) $f(x)=\sqrt{x+2}+4$ $y=\sqrt{-x+2}+4$ $y=\sqrt{-3x+2}+4$ $y=\sqrt{-3(x+3)+2}+4 \Rightarrow \boxed{y=\sqrt{-3x-7}+4}$	1. A Reflection in the y-axis and $x\to -x$ 2. A Horizontal compression by a factor of 1/3. $x\to 3x$ 3. A shifted 3 units left. $x\to x+3$
d) $f(x)=2^x+3$ $y=-2^{-x}-3$ $y=-2^{-\frac{1}{2}x}-3$ $\boxed{y=-2^{-\frac{1}{2}x}-14}$	1. A reflection in both the "x" and "y" axis $x\to -x$ $y\to -$ 2. A horizontal expansion by a factor of 2, $x\to\frac{1}{2}x$ 3. A shifted of 11 units down $y\to y+11$
e) $x^2+(y-1)^2=9$ $(-\frac{1}{2}x)^2+(y-1)^2=9$	1. A reflection in the "y" axis, $x\to -x$ 2. A Horizontal expansion by a factor of 2 and $x\to\frac{1}{2}x$ 3. A vertical compression by a factor of 0.5. $y\to 2y$

$\boxed{\frac{x^2}{4}+(2y-1)^2=9}$

f) $y=\frac{1}{x-1}-3$ $x=\frac{1}{y-1}-3$ $3x=\frac{1}{y-1}-3$ $3x+6=\frac{1}{y-1}-3$ $3x+9=\frac{1}{y-1}$ $\boxed{y=\frac{1}{3x+9}+1}$	1. A reflection in the line $y=x$, $y=f(x)\to x=f(y)$ 2. A Horizontal compression by 1/3, and $x\to 3x$ 3. A shift of 2 units left. $x\to x+2$
g) $y=x^4+x^3-2x+1$ $x=y^4+y^3-2y+1$ $\boxed{x=(y+6)^4+(y+6)^3-2(y-6)+1}$	1. A reflection in the line $y=x$ $y=f(x)\to x=f(y)$ 2. A shift of 6 units down $y\to y+6$
h) $y=\left\|\frac{1}{x-1}\right\|+3$ $\frac{1}{2}y=\left\|\frac{1}{x-1}\right\|+3$ $\frac{1}{2}y=\left\|\frac{1}{4x-1}\right\|+3$ $\frac{1}{2}(y-2)=\left\|\frac{1}{4(x+3)-1}\right\|+3$ $\boxed{y=2\left\|\frac{1}{4x+11}\right\|+8}$	1. A vertical expansion by a factor of 2, $y\to\frac{1}{2}y$ 2. A horizontal compression by a factor of 0.25, $x\to 4x$ 3. A shift of 3 units left and 2 units up. $x\to x+3$, $y\to y-2$
i) $y=3^x$ $y=3^{-x}$ $y=3^{-2x}$ $\boxed{x=3^{-2y}}$ MORE ADVANCED $\log_3 x=-2y\Rightarrow\boxed{y=-\frac{1}{2}\log_3 x}$	1. A horizontal reflection and $x\to -x$ 2. A Horizontal compression by 0.5, and $x\to 2x$ 3. An inverse reflection over the line $y=x$ $y=f(x)\to x=f(y)$

4. If $f(x)=\frac{3x-7}{x+1}$ and $g(x)$ is the inverse of $f(x)$ then determine the value of $g(2)$

Approach 1

$y=\frac{3x-7}{x+1}\Rightarrow x=\frac{3y-7}{y+1}$

$xy+x=3y-7$

$y(x-3)=-7-x$

$y=\frac{-7-x}{x-3}$

$g(x)=-\frac{x+7}{x-3}$

$g(2)=-\frac{2+7}{2-3}=\boxed{9}$

Approach 2

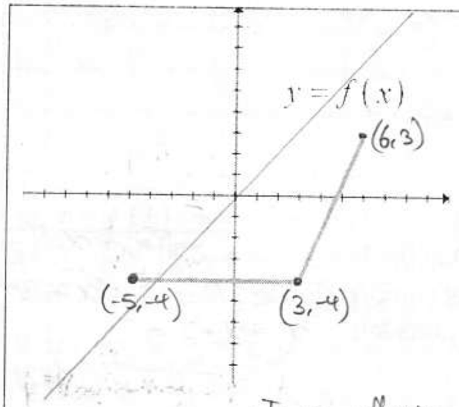
$\frac{3x-7}{x+1}=2$

$2x+2=3x-7$

$x=9$

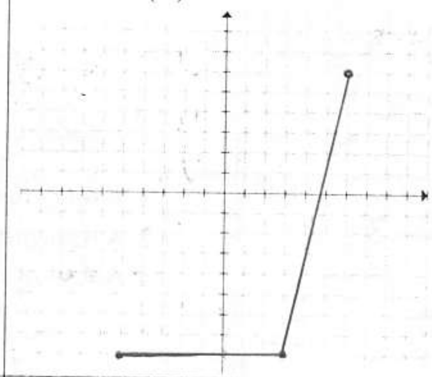
$\boxed{g(2)=9}$

5. Given the graph of $y=f(x)$, draw the graph of the following:



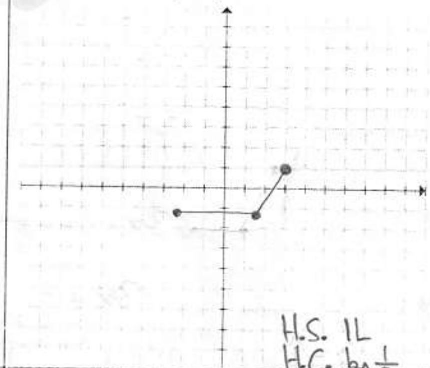
V.E. by 2

i) $y = 2f(x)$



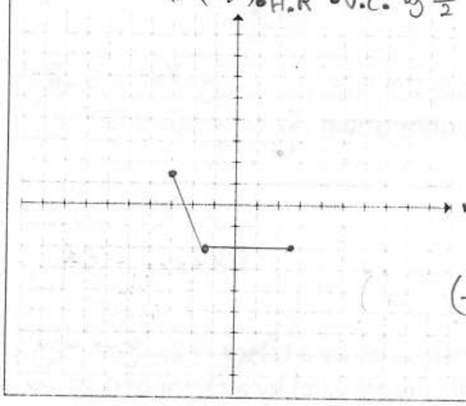
H.C. by $\frac{1}{2}$ V.C. by $\frac{1}{3}$

ii) $y = 0.\bar{3}f(2x)$

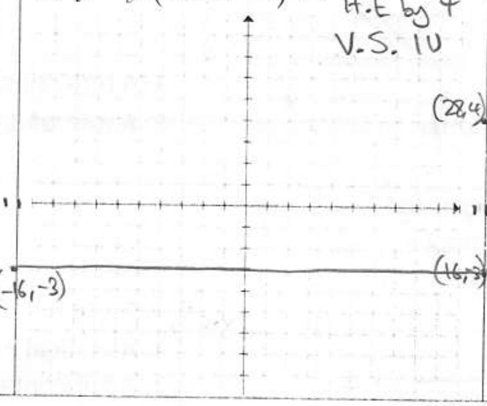


• Inverse reflection

iii) $x = -0.5f(2y)$ • H.C. by $\frac{1}{2}$ • H.R • V.C. by $\frac{1}{2}$

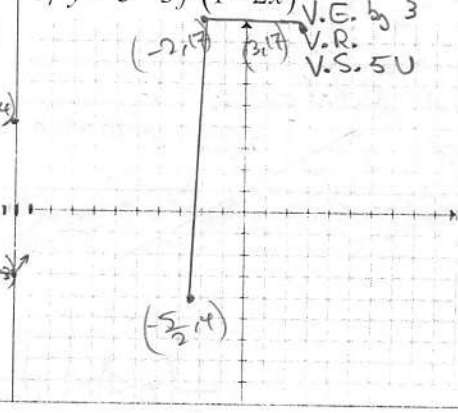


iv) $y = f(0.25x - 1) + 1$ H.S. 1R H.E by 4 V.S. 1U

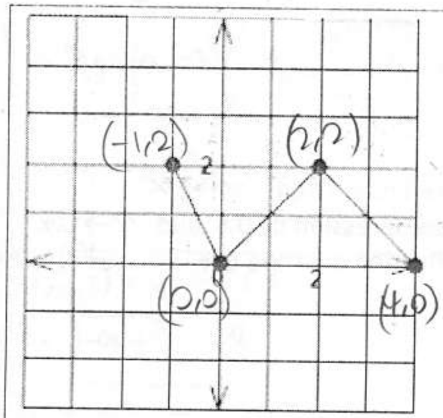


H.S. 1L
H.C. by $\frac{1}{2}$

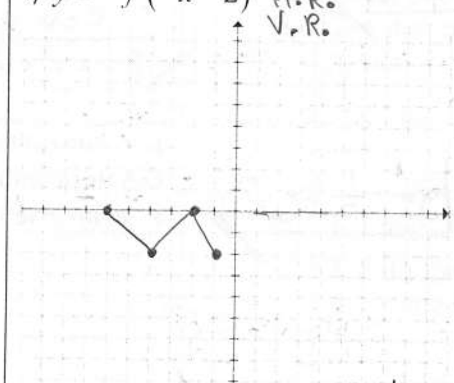
v) $y = 5 - 3f(1 - 2x)$ H.R. V.E. by 3 V.R. V.S. 5U



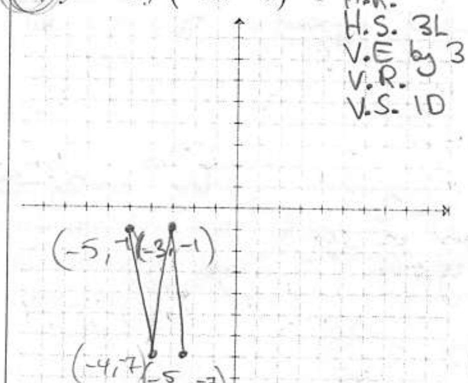
6. Given the graph of $y = f(x)$, draw the graph of the following:



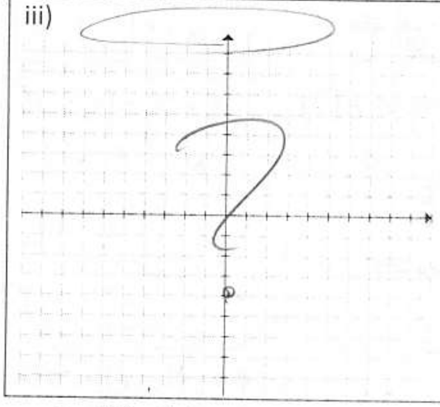
i) $y = -f(-x - 2)$ H.S. 2R H.R. V.R.



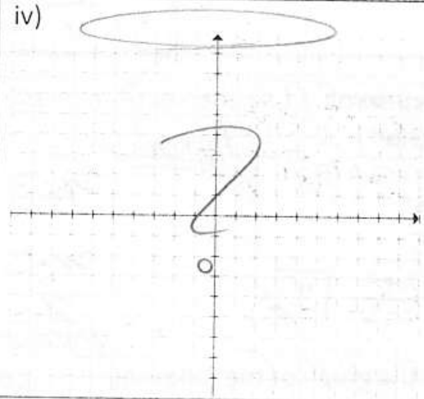
ii) $y = -3f(-2x - 6) - 1$ H.C. by $\frac{1}{2}$ H.R. H.S. 3L V.E by 3 V.R. V.S. 1D



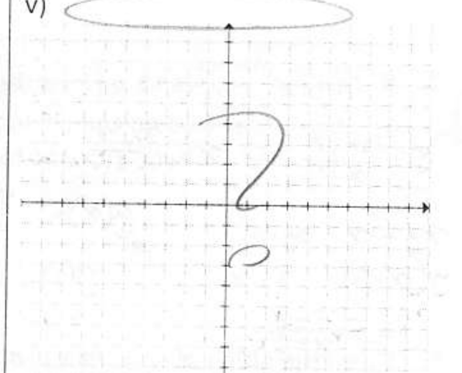
iii)



iv)



v)



7. If (a,b) is a point on the graph of $y = f(x)$, determine the coordinates in each of the following functions:

Restrict range

a) $f(x)=\frac{1}{4}x+5$	b) $y=3(x+7)^2-6$ → vertex: $(-7,-6)$	c) $f(x)=\frac{2}{x-3}$
$y=\frac{1}{4}x+5$ $x=\frac{1}{4}y+5$ $y=4(x-5)$ $y=4x-20$ $\boxed{f^{-1}(x)=4x-20}$	$x=3(y+7)^2-6$ $\pm\sqrt{\frac{x+6}{3}}=y+7$ $y=-7\pm\sqrt{\frac{x+6}{3}}$ right side up $\boxed{f^{-1}(x)=\begin{cases}-7+\sqrt{\frac{x+6}{3}} & \text{if } y\ge -7\\ -7-\sqrt{\frac{x+6}{3}} & \text{if } y\le -7\end{cases}}$ left side down	$x=\frac{2}{y-3}$ $y-3=\frac{2}{x}$ $y=\frac{2}{x}+3$ still a function $\boxed{f^{-1}(x)=\frac{2}{x}+3}$

10. Let $y=mx+b$ be the image when the line $x+3y+11=0$ is reflected across the x-axis. Find the value of $m+b$

V.R. $\rightarrow y \rightarrow -y$ $\quad x+3(-y)+11=0 \Rightarrow x-3y+11=0$

$3y=x+11$

$y=\frac{x+11}{3}$

$y=mx+b$

$\frac{x}{3}+\frac{11}{3}=mx+b$

$m=\frac{1}{3}\quad b=\frac{11}{3}$

$\boxed{m+b=\frac{1}{3}+\frac{11}{3}=4}$

11. For the previous question, what is the reflection is across the line $y=x$. What would the new equation be?

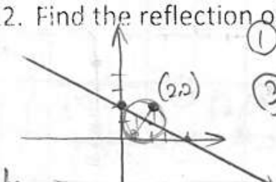
$x+3y+11=0$

Inverse reflection $\quad y+3x+11=0 \Rightarrow \boxed{y=-3x+11}$

$y=mx+b$

$m+b=-3-11=\boxed{-14}$

12. Find the reflection of the point (2,2) in the line $x+2y=4$



silly mistake

① $y=-\frac{x}{2}+2$ ② $(2,2)$ lies on $y=2x+2$ which is perpendicular to $x+2y=4$.

③ The distance d between point (x_1,y_1) and a straight line $ax+by+c=0$ is $d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

④ $d=\frac{|2+4-4|}{\sqrt{1+4}}=\frac{2}{\sqrt5}=\frac{2\sqrt5}{5}$

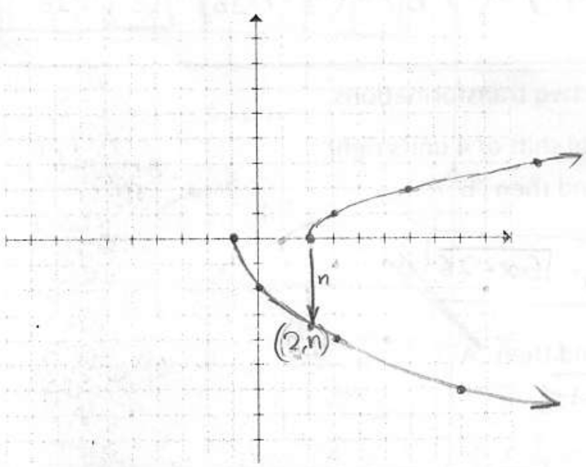
$\frac{|x+4x+4-4|}{\sqrt5}=\frac{5x}{\sqrt5}\Rightarrow\frac{5x}{\sqrt5}=\frac{2}{\sqrt5}\quad x=\frac{2}{5}$

$y=\frac{4}{5}+2=\frac{14}{5}$

⑤ Reflected coordinates $(x_1,y_1)=(x,2x+2)$

Reflected coordinates $=\boxed{\left(\frac{2}{5},\frac{14}{5}\right)}$

13. Draw the graph of $y=-2\sqrt{x+1}$ and $y=\sqrt{x-2}$. For what value(s) of "k" will the graphs of the function $y=-2\sqrt{x+1}$ and $y=\sqrt{x-2}+k$ intersect? (Assume that "x" and "k" are real numbers)



If we shift $y=\sqrt{x-2}$ down by $|n|$ or more units, we'll have a guaranteed intersection between the graphs somewhere.

$y-k=\sqrt{x-2}$

Let's find n.

$n=-2\sqrt{2+1}\Rightarrow n=-2\sqrt3$

$\boxed{k\le-2\sqrt3}$

If k is smaller or equal to $-2\sqrt3$, we'll have a shift of $2\sqrt3$ or more units down.