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Math 12 Honours: HW Section 2.3 Horizontal, Vertical, and Inverse Reflections

H = horizontal
V = vertical
S = shift
R = reflection

1. Indicate the transformation from the function on the left to the function on the right:

a) $y=|x| \rightarrow y=-|x-2|$

Horizontal shift 2 units to the right
vertical reflection over x-axis

b) $y=\sqrt{x} \rightarrow y=\sqrt{3-x}-7$

H.S. 3L
H.R. over y-axis
V.S. 7D

c) $y=3x+2 \rightarrow y=-3x-2$

Vertical transformation over the x-axis

V.T.

d) $y=x^2 \rightarrow y=-x^2-2x-4$

$y=-(x+1)^2-3$
$-y=(x+1)^2+3$

H.S. 1L
V.S. 3U
V.R over x-axis

e) $y=2^{3x+1} \rightarrow x=2^{3y+1}$

Inverse transformation over $y=x$

I.T.

f) $y=\frac{1}{x} \rightarrow y=\frac{-1}{-x+5}$

$-y=\frac{1}{-x+5}$

DO H.S. before H.R.

H.S. 5L
H.R. over y-axis
V.R. over x-axis

2. Given each equation for $y=f(x)$, indicate the new equation after each transformation in the order stated:

a) $f(x)=2x+3$ $f(x)=-2x+3$ $f(x)=-2(x-3)+3$ $f(x)=-2(x-3)+5$	1. A horizontal reflection over the Y-axis $x \to -x$ 2. A shift of 3 units right $x \to x-3$ 3. A shift of 2 units up $y \to y-2$
b) $f(x)=\frac{2}{3}(x-1)^2+1$ $f(x)=-\frac{2}{3}(x-1)^2-1$ $f(x)=-\frac{2}{3}(x+1)^2-1$ $f(x)=-\frac{2}{3}(x+1)^2-7$	1. A vertical reflection over the X-axis $y \to -y$ 2. A shift of 2 units left $x \to x+2$ 3. A shift of 6 units down $y \to y+6$
c) $f(x)=\sqrt{x+2}-3$ $x=\sqrt{y+2}-3$ $x+4=\sqrt{y+2}-3$ $x+4=\sqrt{y-4}-3$	1. A reflection over the Y=x line $y=f(x) \to x=f(y)$ 2. A shift of 4 units left $x \to x+4$ 3. A shift of 6 units up $y \to y-6$
d) $f(x)=5^x-1$ $f(x)=-5^{-x}+1$ $f(x)=-5^{-x-3}+1$ $f(x)=-5^{-x-3}-10$	1. A reflection in both the "x" and "y" axis 2.A shift of 3 units right 3. A shift of 11 units down
e) $x^2+y^2=9$ $(x-3)^2+y^2=9$ $(x-3)^2+(y-2)^2=9$ $(-x-3)^2+(y-2)^2=9$	1. A shift of 3 units right 2. A shift of 2 units up 3. A reflection over the "y" axis, $x \to -x$

DON'T CHANGE -3 to 3!!!
ONLY $x \to -x$

f) $y=\frac{1}{x+2}-3$ $y=\frac{1}{x+4}-3$ $y=\frac{1}{x+4}-9$ $\boxed{x=\frac{1}{y+4}-9}$	1. A shift of 2 units left, 2. A shift of 6 units down 3. A reflection in the line $y=x$,
g) $y=x^4+x^3-2x+1$ $x=y^4+y^3-2y+1$ $\boxed{x=(y+6)^4+(y+6)^3-2(y+6)+1}$	1. A reflection in the line $y=x$ 2. A shift of 6 units down $x \Leftrightarrow y$ $y \to y+6$
h) $y=\left\|\frac{1}{x-1}\right\|+3$ $y=\left\|\frac{1}{-x-1}\right\|+3$ $y=\left\|\frac{1}{-(x-4)-1}\right\|+14$ $\boxed{y=-\left\|\frac{1}{-x+3}\right\|+14}$	1. A reflection in the "y" axis 2. A shift of 4 units right 3.A shift of 11 units up 4. A reflection over the x-axis.
i) $y=x^3-3x$ $y=-x^3+3x$ $\boxed{x=-y^3+3y}$	1. A horizontal reflection over the Y-axis 2. Then an inverse reflection over the line $y=x$

3. Given that the coordinates (a,b) are on the function $y=f(x)$, find the new coordinates for each function after the transformation:

a) $y=f(-x)$ $\boxed{(-a,b)}$	b) $y=f(-x+3)$ H.S. 3L $(a,b)\to(a-3,b)$ H.R. over y-axis $(a-3,b)\to\boxed{(3-a,b)}$
c) $y=-f(x+2)$ H.S. 2L $(a-2,b)$ V.R. over x-axis $\boxed{(a-2,-b)}$	d) $y=f(-x)+2$ H.R. over y-axis $(-a,b)$ V.S. 2U $\boxed{(-a,b+2)}$
e) $y=-f(-x)+3$ H.R. over y axis $(-a,b)$ V.S. 3D $(-a,b-3)$ V.R. over x-axis $\boxed{(-a,-b+3)}$	f) $-x+1=f(2-y)$ Inverse R. $x=f(x)$ (b,a) H.S. 1L $x\to x+1$ $x+1=f(x)$ $(b-1,a)$ H.R. $x\to -x$ $-x+1=f(x)$ $(1-b,a)$ V.S. 2D $y\to y+2$ $-x+1=f(y+2)$ $(1-b,a-2)$ V.R. $y\to -y$ $-x+1=f(-y+2)$ $\boxed{(1-b,2-a)}$

Tip: in circling method, do shifts and the reflections

g) $y=-f(-x+7)-5$

$y=f(x) \to y=-f(-x+7)-5$

$x \to x+7$ $\quad y=f(x+7)$ $\quad$ H.S. 7L $\quad (a-7, b)$

$x \to -x$ $\quad y=f(-x+7)$ $\quad$ H.R. $\quad (-a+7, b)$

$y=f(-x+7)-5$ $\quad$ V.S. 5D $\quad (-a+7, b-5)$

$-y=f(-x+7)-5$ $\quad$ V.R. $\quad \boxed{(-a+7, 5-b)}$

h) $4-x=f(3-y)$ $\quad (a,b)$

Inverse reflection $\quad x=f(y)$ $\quad (b,a)$

V.S. 3D $\quad x=f(y+3)$ $\quad (b,a-3)$

V.R. $\quad x=f(-y+3)$ $\quad (b,3-a)$

H.S. 4L $\quad x+4=f(-y+3)$ $\quad (b-4,3-a)$

H.R. $\quad -x+4=f(-y+3)$ $\quad \boxed{(4-b,3-a)}$

i) $-y=f(-x+3)-2$

$y=f(x)$ $\quad (a,b)$

H.S. 3L $\quad y=f(x+3)$ $\quad (a-3,b)$

H.R $\quad y=f(-x+3)$ $\quad (3-a,b)$

V.S. 2D $\quad y=f(-x+3)-2$ $\quad (3-a,b-2)$

V.R. $\quad -y=f(-x+3)-2$ $\quad \boxed{(3-a,2-b)}$

NOTE: we can tell what the new coordinates look like roughly by simply looking at what replaces "x" & "y" in $y=f(x)$

j) $11+x=f(-y+1)+2$ = $x=f(-y+1)-9$

$y=f(x)$

Inverse reflection $\quad x=f(y)$ $\quad (b,a)$

V.S. 1D $\quad x=f(y+1)$ $\quad (b,a-1)$

V.R. $\quad x=f(-y+1)$ $\quad (b,1-a)$

H.S. 9L $\quad x=f(-y+1)-9$ $\quad \boxed{(b-9,1-a)}$

V.S. NOT H.S. I AM DEALING WITH Y VARIABLE.

k) $y=f^{-1}(x)+2$ $\quad (a,b)$

$y=f(x)$

Inverse function $\quad y=f^{-1}(x)$ or $x=f(y)$ $\quad (b,a)$

H.S. 2R $\quad x=f(y)+2$ $\quad \boxed{(b+2,a)}$

L) $y-2=|f(x+1)|$

$y=f(x) \to y-2=|f(x+1)|$ $\quad (a,b)$

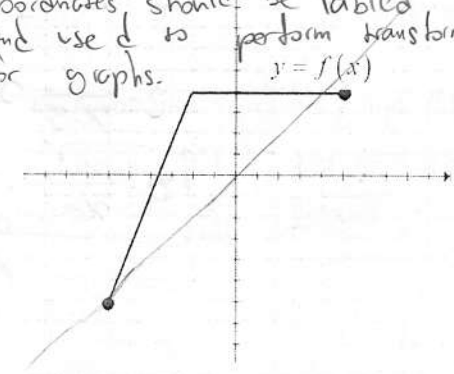
$y=f(x+1)$ $\quad$ H.S. 1L $\quad (a-1,b)$

$y-2=f(x+1)$ $\quad$ V.S. 2U $\quad (a-1,b+2)$

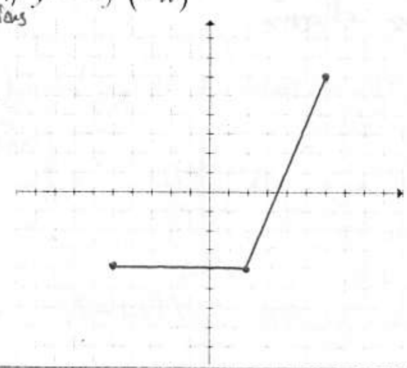
$y-2=|f(x+1)|$ absolute value transformation $\quad \boxed{(a-1,|b+2|)}$

4. Given the graph of $y=f(x)$, draw the resulting image after each transformation:

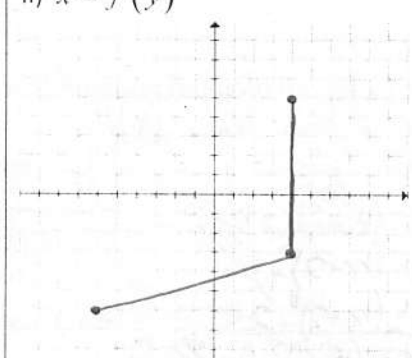
Coordinates should be labled and used to perform transformations for graphs.

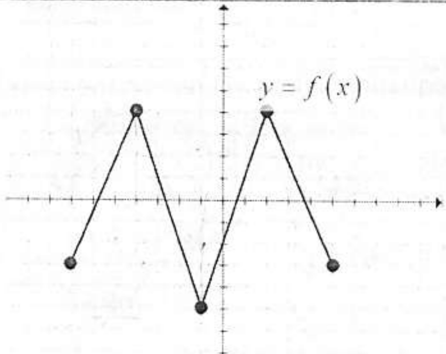


i) $y=-f(-x)$

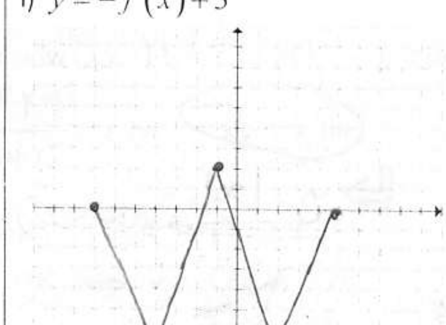


ii) $x=f(y)$

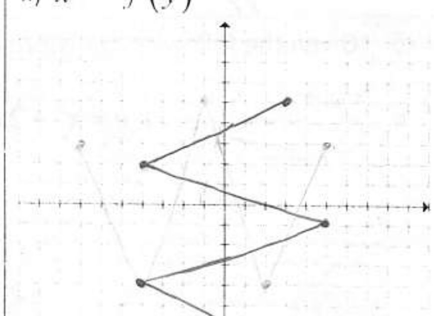


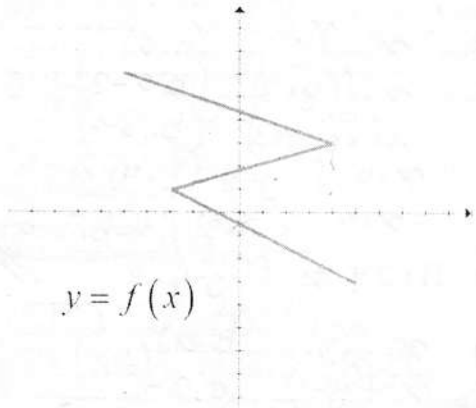


i) $y=-f(x)+3$



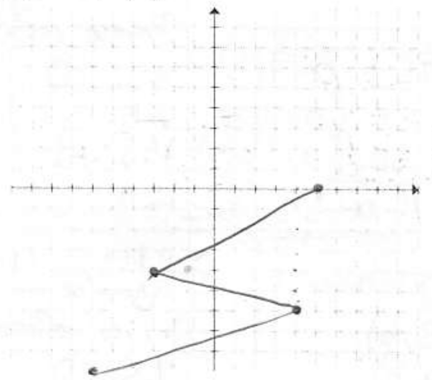
ii) $x=-f(y)$



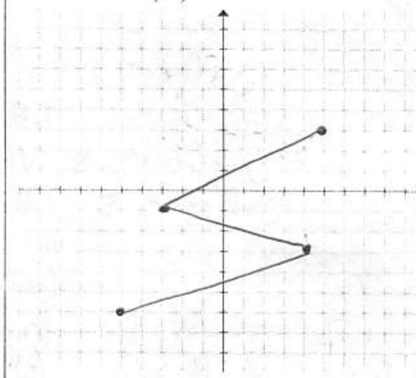


$y = f(x)$

i) $y = -f(x) + 3$

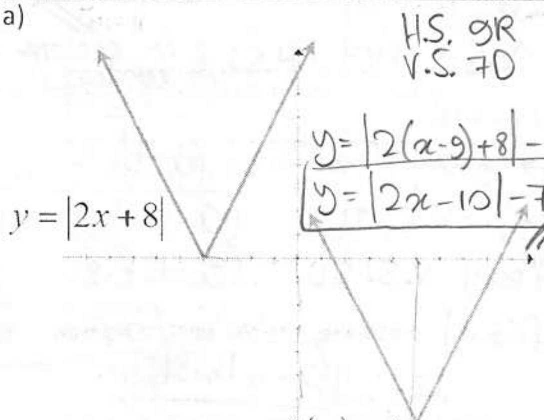


ii) $x = -f(y)$



5. Given the graph of $y = f(x)$ and the graph after transformation, what is the equation of the new graph?

a)



H.S. 9R $x \to x-9$
V.S. 7D $y \to y+7$

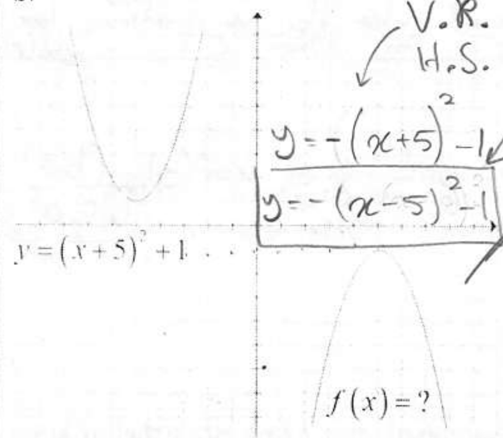
$y = |2x+8|$

$y = |2(x-9)+8| - 7$

$\boxed{y = |2x-10| - 7}$

$f(x) = ?$

b.



V.R. $y \to -y$
H.S. 10R $x \to x-10$

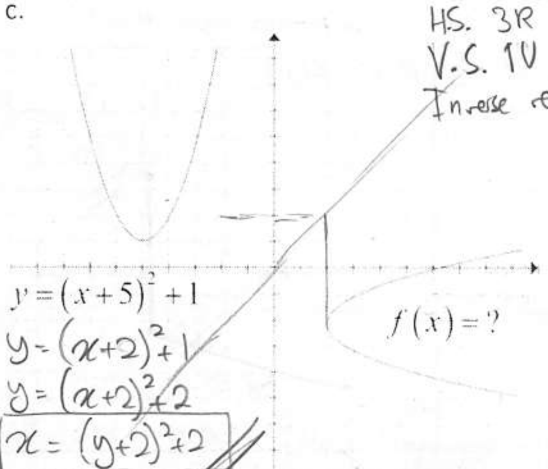
$y = (x+5)^2 + 1$

$y = -(x+5)^2 - 1$

$\boxed{y = -(x-5)^2 - 1}$

$f(x) = ?$

c.



H.S. 3R $x \to x-3$
V.S. 1U $y \to y-1$
Inverse reflection

$y = (x+5)^2 + 1$

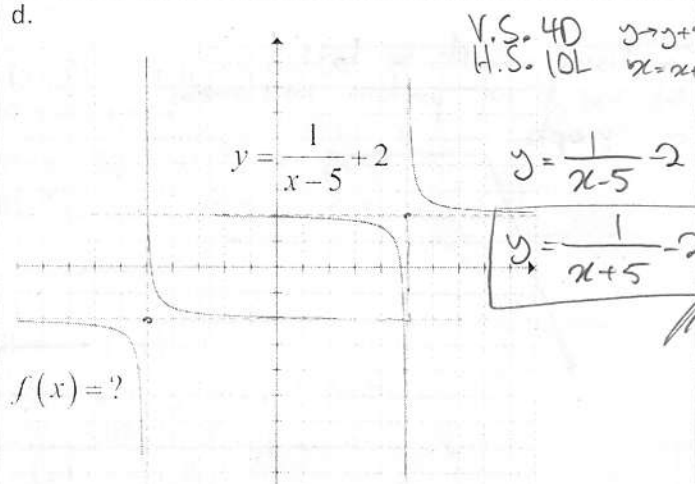
$y = (x+2)^2 + 1$

$y = (x+2)^2 + 2$

$\boxed{x = (y+2)^2 + 2}$

$f(x) = ?$

d.



V.S. 4D $y \to y+4$
H.S. 10L $x = x+10$

$y = \frac{1}{x-5} + 2$

$y = \frac{1}{x-5} - 2$

$\boxed{y = \frac{1}{x+5} - 2}$

$f(x) = ?$

6. Given the following transformation, $y = f(x) \to y = f(-x)$, which equation below will remain the same?

i) $y = x^2$ ii) $y = x^3 + 2x^2$ iii) $y = \sqrt{x^2}$ iv) $y = \frac{1}{2x+3}$ vi) $y = |3(2^x)|$

↳ $y = |x|$

7. Given that $y = x^3 - 2x^2 + 3x + 4$, what is the equation of the resulting graph after an inverse reflection over the line $y = x$?

$y = f(x) \longrightarrow x = f(y)$

$y = x^3 - 2x^2 + 3x + 4 \longrightarrow \boxed{x = y^3 - 2y^2 + 3y + 4}$

8. The domain and range of $y = f(x)$ is $D: x > 7$ and $R: -4 < y < 10$. What is the domain and range of

i) $y = f(-x)$

$\boxed{D: x < -7 \quad R: -4 < y < 10}$

Domain changes.

$y = f(-x)$ $y = f(x)$ -7 7

ii) $y = -f(x)$

$\boxed{D: x > 7 \quad R: -10 < y < 4}$

Range changes

iii) $y = -f(-x)$

$\boxed{D: x < -7 \quad R: -10 < y < 4}$

Both Domain & Range change

iv) $x = f(y)$

$\boxed{D: -4 < x < 10 \quad R: y > 7}$

Domain & range swap places.

9. Given that $f(x) = 3x + 2$ and $f_2(x) = -3x + 2$. What are all the transformation required for $f(x)$ to become $f_2(x)$?

$f(x)$ would need to undergo a horizontal reflection in order to become $f_2(x)$.

10. Given that $f(x) = 2^x$ and $f_2(x) = 0.5^x$. What are all the transformation required for $f(x)$ to become $f_2(x)$?

$f(x) = 2^x$

$f_2(x) = \left(\frac{1}{2}\right)^x = 2^{-x}$

$\therefore$ $f(x)$ would need to undergo a horizontal reflection in order to become $f_2(x)$

11. Given that $f(x) = \sqrt{x}$ and $f_2(x) = -\sqrt{-x+3} + 4$. What are all the transformation required for $f(x)$ to become $f_2(x)$? List them in order.

$y = \sqrt{x} \longrightarrow y = -\sqrt{-x+3} + 4$

$y = \sqrt{x}$

H.S. 3L $\quad y = \sqrt{x+3}$

H.R. $\quad y = \sqrt{-x+3}$

V.R. $\quad y = -\sqrt{-x+3}$

V.S. 4U $\quad y = -\sqrt{-x+3} + 4$

$f(x)$ would need to undergo:
- a horizontal shift of 3 left
- a horizontal reflection
- V.R.
- V.S. 4U

in order to become $f_2(x)$.

12. If the function $f(x) = x^2 + 8x + 16$ is shifted 4 units up, 3 right, and reflected over the x-axis, the equation is now: $f(x) = a(x+b)^2 + c$, what is the value of $a + b + c$?

$f(x) = x^2 + 8x + 16 = (x+4)^2$

V.S. 4U $\quad y = (x+4)^2 + 4$

H.S. 3R $\quad y = (x - 3 + 4)^2 + 4 = (x+1)^2 + 4$

V.R. $\quad y = (x+1)^2 + 4 \Rightarrow y = (x-1)^2 + 4$

$y = a(x+b)^2 + c \longrightarrow \left.\begin{array}{l} a = 1 \\ b = -1 \\ c = 4 \end{array}\right\} \; a + b + c = \boxed{4}$

13. Given that $f(x)=\frac{1}{x}+2$ and $f_2(x)=-\frac{1}{x+3}+4$. What are all the transformation required for $f(x)$ to become $f_2(x)$? List them in order.

Approach 1

H.S. 3L	$x \to x+3$	$y=\frac{1}{x+3}+2$	$(a-3, b)$
V.R.	$y \to -y$	$y=-\frac{1}{x+3}-2$	$(a-3, -b)$
V.S. 6U	$y \to y-6$	$y=-\frac{1}{x+3}+4$	$(a-3, -b+6)$

Approach 2

H.S. 3L	$x \to x+3$	$y=\frac{1}{x+3}+2$	$(a-3, b)$
V.S. 6D	$y \to y+6$	$y=\frac{1}{x+3}-4$	$(a-3, b-6)$
V.R.	$y \to -y$	$y=-\frac{1}{x+3}+4$	$(a-3, 6-b)$

14. Given that $f(x)=3x+2$ and $f_2(y)=-\frac{1}{3y+3}+2$. What are all the transformation required for $f(x)$ to become $f_2(y)$? List them in order.

$y=3x+2$

- V.S. 1U $\quad y \to y-1 \quad y=3x+3$
- reciprocal function $\quad f(x) \to \frac{1}{f(x)} \quad y=\frac{1}{3x+3}$
- Inverse reflection $\quad y=f(x) \Rightarrow x=f(y) \quad x=\frac{1}{3y+3}$
- H.R. $\quad x \to -x \quad x=-\frac{1}{3y+3}$
- H.S. 2R $\quad x \to x-2 \quad x=-\frac{1}{3y+3}+2$

Steps: V.S. 1U $(a, b+1)$; reciprocal $\left(a, \frac{1}{b+1}\right)$; Inverse $\left(\frac{1}{b+1}, a\right)$; H.R. $\left(\frac{1}{-b-1}, a\right)$; H.S. 2R $\left(\frac{1}{-b-1}+2, a\right)$

Important!! Do reciprocal before inverse b/c reciprocal only chang $f(x)$, no

15. If $f(x)=\frac{4x+1}{3}$, what is the value of $\left(f^{-1}(1)\right)^{-1}$?

$y=\frac{4x+1}{3}$

$x=\frac{4y+1}{3} \Rightarrow 3x=4y+1 \Rightarrow y=\frac{3x-1}{4}$

$f^{-1}(x)=\frac{3x-1}{4}$

$f^{-1}(1)=\frac{3-1}{4}=\frac{1}{2}$

$\left(f^{-1}(1)\right)^{-1}=\left(\frac{1}{2}\right)^{-1}=\boxed{2}$

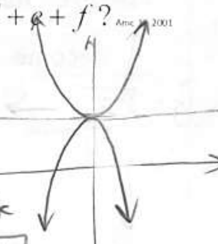
16. If the domain and range of $f(x)$ is $-2 \le x \le 7$, $4 \le y < 11$, *and* $y > 11$, what is the domain and range of $y=-f(-x+4)-3$?

$y=f(x) \to y=-f(-x+4)-3$

		Domain	Range
- H.S. 4L $\quad x \to x+4$	$y=f(x+4)$	$-6 \le x \le 3$	$4 \le y < 11$ and $y > 11$
- H.R. $\quad x \to -x$	$y=f(-x+4)$	$-3 \le x \le 6$	$4 \le y < 11$ and $y > 11$
- V.R. $\quad y \to -y$	$y=-f(-x+4)$	$-3 \le x \le 6$	$-11 < y \le -4$ and $y < -11$
- V.S. 3D $\quad y \to y+3$	$y=-f(-x+4)-3$	$-3 \le x \le 6$	$-14 < y \le -7$ and $y < -14$

17. A parabola with equation $y=ax^2+bx+c$ and vertex (h,k) is reflected about the line $y=k$. This results in the parabola with equation $y=dx^2+ex+f$. Which of the following equals $a+b+c+d+e+f$? AMC 2001

a) 2b b) 2c c) 2a+2b d) 2h e) 2k



$y=ax^2+bx+c \Rightarrow a(x-h)^2+k = ax^2 \boxed{-2ah}x+(ah^2+k)$

$y=dx^2+ex+f \Rightarrow -a(x-h)^2+k = -ax^2 \boxed{+2ah}x+(-ah^2+k)$

$a+d=0$

$b+e=0$

$c+f=ah^2-ah^2+k+k=2k$

$a+b+c+d+e+f=\boxed{2k}$

18. What are all ordered pairs of numbers (x,y) which satisfy: CNML v2 4-6

$x^2-xy+y^2=13$ and $x-xy+y=-5$?

$x+y=xy-5$

$x^2+2xy+y^2=x^2y^2-10xy+25$

subtract

$x^2+y^2=x^2y^2-12xy+25$

$\cancel{x^2}+\cancel{y^2}-\cancel{x^2}+xy-\cancel{y^2}=x^2y^2-12xy+25-13$

$x^2y^2-13xy+12=0$

$(xy-1)(xy-12)=0 \Rightarrow xy=1$ OR $xy=12$

$x=\frac{1}{y}$

$\frac{1}{y}-1+y=-5$

$\frac{1}{y}+y+4=0$

$1+y^2+4y=0$

$y=\frac{-4\pm\sqrt{16-4}}{2}$

$y=\frac{-4\pm2\sqrt{3}}{2}$

$\boxed{y=-2\pm\sqrt{3}}$

$x=\frac{1}{-2\pm\sqrt{3}}$

$\left(\frac{1}{y}\right)^2-1+y^2=13$

$\frac{1}{y^2}-14+y^2=0$

$1-14y^2+y^4=0$

$y^2=\frac{14\pm\sqrt{14^2-4}}{2}$

Garanceel

$x=\frac{12}{y}$

$\frac{12}{y}-12+y=-5$

$y-7+\frac{12}{y}=0$

$y^2-7y+12=0$

$(y-3)(y-4)=0$

$\boxed{y_1=3 \quad y_2=4, \quad x_1=4 \quad x_2=3}$

FINAL