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Pre Calculus 12: Section ~~1.2~~ 2.2 Horizontal and Vertical Translations

1. Indicate the transformation from the function on the left to the function on the right:

a) $y=|x| \to y=|x-2|+4$

Horizontal Shift/transformation 2 units to the right
Vertical shift/transformation 4 units up.

b) $y=\sqrt{x} \to y=\sqrt{x+3}-7$

H.S. 3L
V.S. 7D

c) $y=3x+2 \to y=3x+8$

$y=3(x+2)+2$

V.S. 6U OR H.S. 2L

d) $y=x^2 \to y=x^2+6x+12$

$=(x+3)^2+3$

H.S. 3L
V.S. 3U

e) $y=x^3 \to y=x^3+3x^2+3x+1$

$=(x+1)^3$

H.S. 1L

f) $y=\frac{1}{x} \to y=\frac{1}{x+5}+3$

H.S. 5L V.S. 3U

2. Given that the coordinates (a,b) are on the function $y=f(x)$, find the new coordinates for each function after the transformation:

i) $y=f(x-3)+2$ $(a+3,\ b+2)$	ii) $y-5=f(x+1)+2$ $\Rightarrow y=f(x+1)+7$ $(a-1,\ b+7)$
iii) $y=f(x+7)-11$ $(a-7,\ b-11)$	iv) $y-4=f(x-5)+3$ $\Rightarrow y=f(x-5)+7$ $(a+5,\ b+7)$

3. Given each equation for $y=f(x)$, indicate the new equation after each translation:

a. $f(x)=3x-5$ — A horizontal shift of 3 units right and 2 units up

$f(x)=3(x-3)-5+2=3(x-3)-3=\boxed{3x-12}$

b. $f(x)=2x^2+3$ — A horizontal shift of 5 units left and 8 units up

$f(x)=2(x+5)^2+3+8=2(x+5)^2+11=\boxed{2x^2+20x+66}$

c. $f(x)=\sqrt{x-5}+1$ — A horizontal shift of 7 units right and 6 units down

$f(x)=\sqrt{(x-7)-5}+1-6=\boxed{\sqrt{x-12}-5}$

d. $f(x)=3^x+2$ — A horizontal shift of 11 units left and 2 units down

$f(x)=3^{(x+11)}+2-2=\boxed{3^{x+11}}$

e. $x^2+y^2=16$ — A horizontal shift of 4 units right and 8 units up

$\boxed{(x-4)^2+(y-8)^2=16}$

equation of a circle:
$(x-h)^2+(y-k)^2=r^2$
$(h,k)=$ coordinates of centre

f. $f(x)=\dfrac{1}{x-3}$ A horizontal shift of 2 units left and 6 units down

$f(x)=\dfrac{1}{(x+2)-3}-6=\boxed{\dfrac{1}{x-1}-6}$

4. Given that the coordinates of (2,3) is transformed to (8,4) from $y=f(x)\rightarrow y=f(x-a)+b$, what is the value of a + b?

$(2,3)\rightarrow(8,4)$ $y=f(x)\rightarrow y=f(x-6)+1$

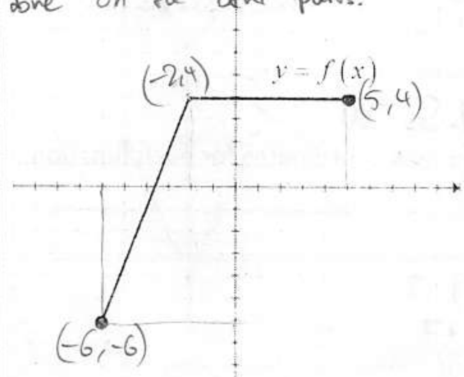
H.S. 6R
V.S. 1U

$a=6$
$b=1$ $\Rightarrow \boxed{a+b=1+6=7}$

5. Given the graph of $y=f(x)$, draw the resulting image after each transformation:

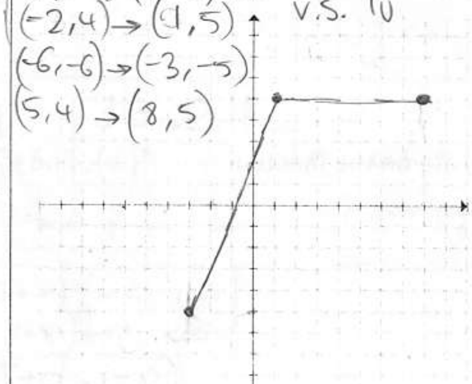
Note, you should always lable coordinates and use them to make transformations.

Please excuse that this is not done on the other parts.

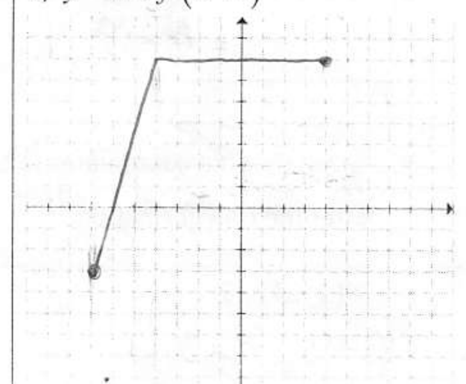


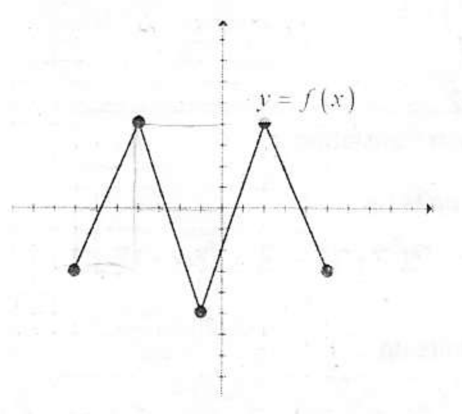
i) $y=f(x-3)+1$ H.S. 3R V.S. 1U

$(-2,4)\rightarrow(1,5)$
$(-6,-6)\rightarrow(-3,-5)$
$(5,4)\rightarrow(8,5)$

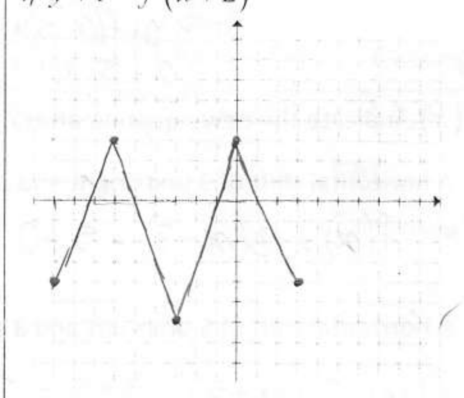


ii) $y-3=f(x+1)$

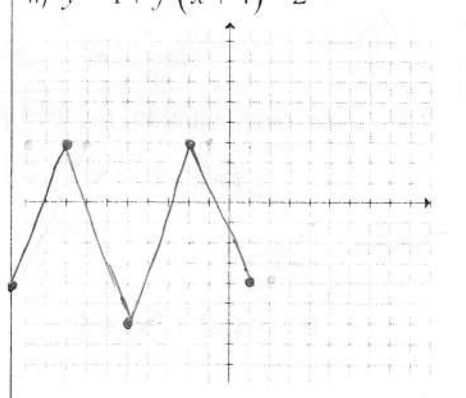


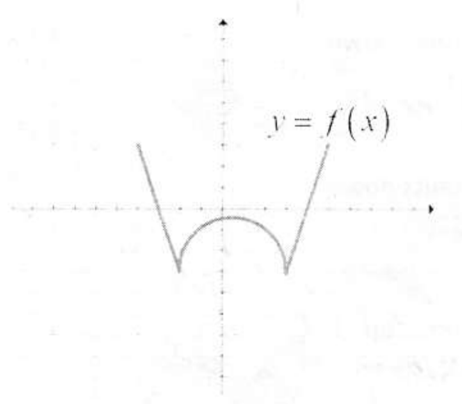


i) $y+1=f(x+2)$

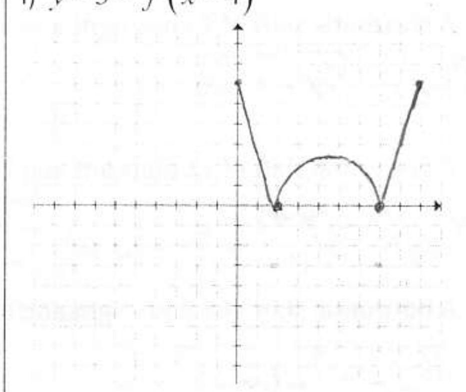


ii) $y=1+f(x+4)-2$

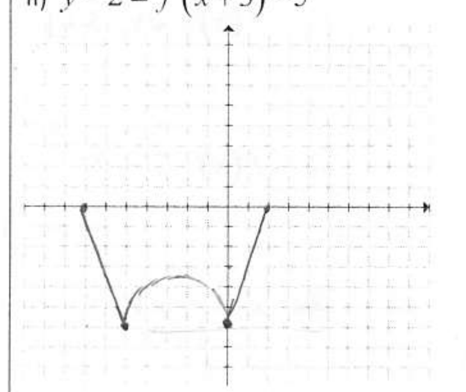




i) $y-3=f(x-4)$



ii) $y-2=f(x+3)-5$



flip over y-axis

6. Given the following transformation, $y = f(x) \to y = f(-x)$, which equation below will remain the same?

i) $y = x^2$ ii) $y = x^3 + 2x^2$ iii) $y = \sqrt{x^2}$ iv) $y = \dfrac{1}{2x+3}$ vi) $y = \left|3\left(2^x\right)\right|$

$y = x^2$ is symmetrical along the x-axis.

$$\left.\begin{aligned} f(x) &= x^2 \\ f(-x) &= (-x)^2 = x^2 \end{aligned}\right\} \text{same}$$

7. Solve the following equations algebraically for "x". Then use the grid on the left to graph each side of the equation as a separate function. Ie: Y1 is the left side and Y2 is the right side of the equation. Use the graph to find the intersection points: Indicate all the extraneous roots. Only use Graphing Technology to check:

a) $|x-2|+1 = \dfrac{3x+3}{5}$

case 1

$x - 2 + 1 = \dfrac{3x+3}{5} \Rightarrow 5(x-1) = 3x+3$

$5x - 5 = 3x + 3$

$2x = 8$

$\boxed{x = 4}$

$(x, y) = (4, 3)$

case 2

$-x + 2 + 1 = \dfrac{3x+3}{5}$

$5(3 - x) = 3x + 3$

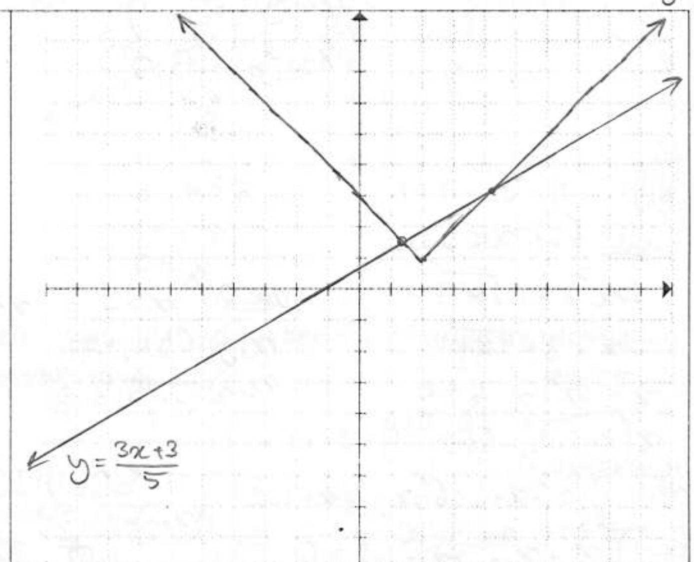
$15 - 5x = 3x + 3$

$8x = 12$

$\boxed{x = \frac{3}{2}}$

$(x, y) = \left(\frac{3}{2}, \frac{3}{2}\right)$

Both solutions are valid.



b) $\dfrac{1}{x-1} + 2 = 4x - 2$

$\dfrac{1}{x-1} + 4 = 4x$

$\dfrac{1}{x-1} = 4(x-1)$

$4(x-1)^2 = 1$

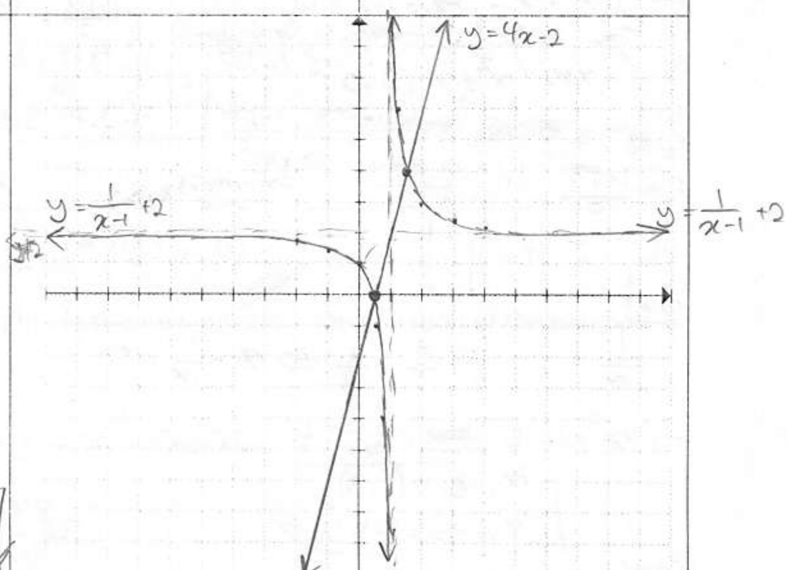
$4x^2 - 8x + 4 = 1$

$4x^2 - 8x + 3 = 0$

$x = \dfrac{8 \pm \sqrt{64 - 4(4 \cdot 3)}}{8} = \dfrac{8 \pm \sqrt{64-48}}{8} = \dfrac{8 \pm 4}{8}$ $\boxed{x_1 = \frac{3}{2}, \; x_2 = \frac{1}{2}}$

$(x_1, y) = \left(\frac{3}{2}, 4\right)$

$(x_2, y_2) = \left(\frac{1}{2}, 0\right)$



c) $\sqrt{x-1}=|x-3|$

Case 1

$\sqrt{x-1}=x-3$

$x-1=(x-3)^2 \Rightarrow x-1=x^2-6x+9 \Rightarrow x^2-7x+10$

$(x-2)(x-5)=0$

$x_1=2$

$x_2=5$

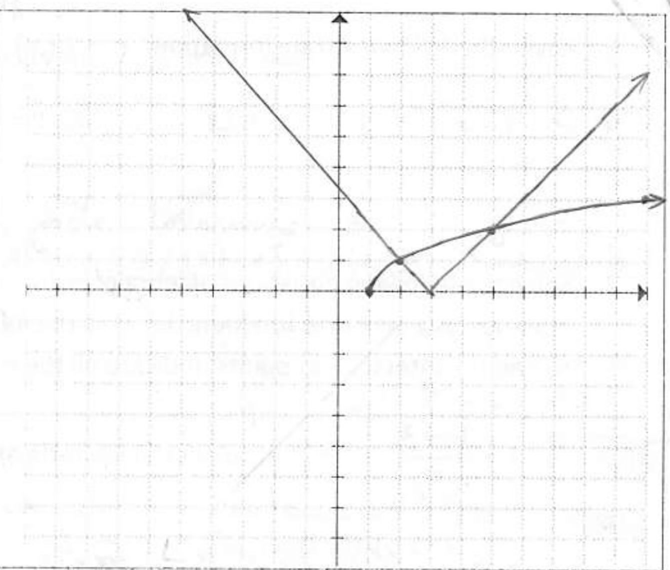
Case 2

$\sqrt{x-1}=3-x$

leads to same answers

$(x_1,y_1)=(2,1)$

$(x_2,y_2)=(5,2)$



MISSED FACTORING!!

d) $|x^2-4|=\sqrt{x+3}+1$

$\rightarrow x^2-4=0$

$x^2=4$

$x=\pm 2$

Case 1 $(-2\le x<2)$

$-x^2+4=\sqrt{x+3}+1$

$-x^2+3=\sqrt{x+3}$

$x^4-6x^2-x+6=0$

$x(x^3-1)-6(x^2-1)=0$

$x(x-1)(x^2+x+1)-6(x-1)(x+1)=0$

$(x-1)(x^3+x^2-5x-6)=0$

If we plug in -2, it works! so factor out $x-2$ using long division.

$(x-1)(x+2)(x^2-x-3)=0$

$x=\frac{1\pm\sqrt{1+4(-3)}}{2}=\frac{1\pm\sqrt{13}}{2}$

$r_1=1$

~~$r_2=-2$~~ rej. extraneous

~~$r_3=\frac{1+\sqrt{13}}{2}$~~ → rej extraneous since $r_3>2$

$r_4=\frac{1-\sqrt{13}}{2}$

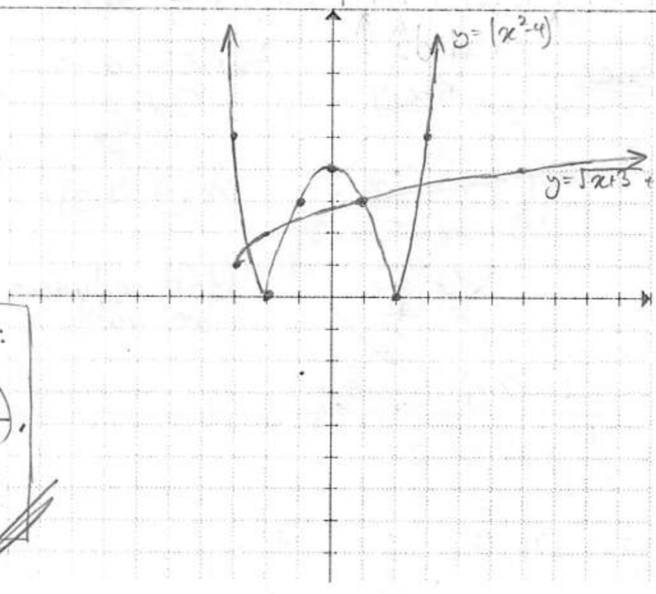
Case 2 $(x\ge 2$ or $x<-2)$

$(x_1,y_1)=(-2.4,1.77)$

$(x_2,y_2)=(2.79,3.39)$

SOLUTIONS:

$(x,y)=(1,3),$
$\left(\frac{1-\sqrt{13}}{2},\frac{1+\sqrt{13}}{2}\right),$
$(-2.4,1.77),$
$(2.79,3.39)$



e) $\frac{1}{x}+2=\left|x+\frac{3}{2}\right|+\frac{1}{2}$

Case 1

$\frac{1}{x}+2=x+\frac{3}{2}+\frac{1}{2} \Rightarrow x-\frac{1}{x}=0$

$\Rightarrow x^2-1=0 \Rightarrow x=\pm 1$

$(x_1,y_1)=(1,3)$

$(x_2,y_2)=(-1,1)$

Case 2

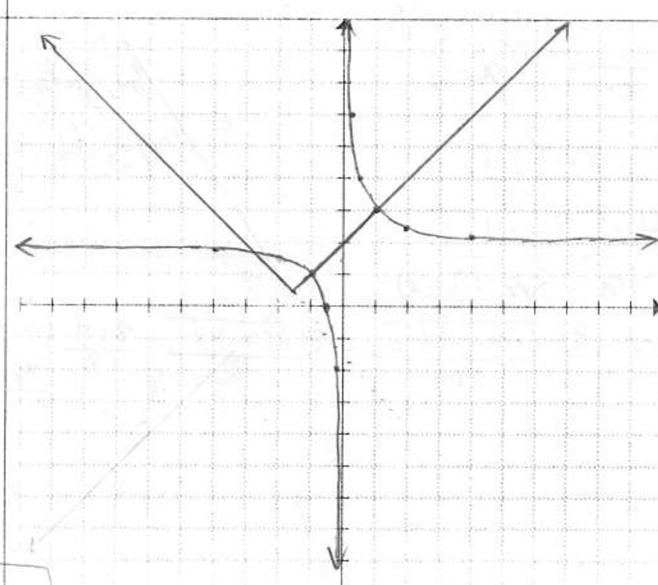
$\frac{1}{x}+2=-x-\frac{3}{2}+\frac{1}{2} \Rightarrow x+3+\frac{1}{x}=0$

$\Rightarrow x^2+3x+1=0$

$x=\frac{-3\pm\sqrt{9-4(1)}}{2}=\frac{-3\pm\sqrt{5}}{2}$

~~$(x_3,y_3)=\left(\frac{-3+\sqrt{5}}{2},\frac{2\sqrt{5}-4}{-3+\sqrt{5}}\right)$~~ rej. extraneous root.

$(x_4,y_4)=\left(\frac{-3-\sqrt{5}}{2},\frac{4+2\sqrt{5}}{3+\sqrt{5}}\right)$



f) $-|0.5x-1|-1=2^{x-1}-3$

case 1

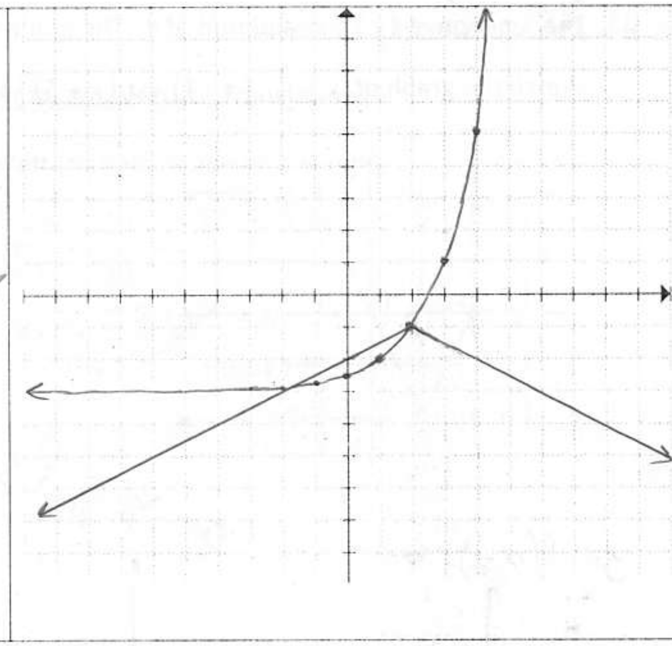
$-\frac{1}{2}x+\cancel{1}-\cancel{1}=2^{x-1}-3 \Rightarrow -x=2^{x}-6$

case 2

$\frac{1}{2}x-1-1=2^{x-1}-3 \Rightarrow x=2^{x}-2$

$\boxed{(x_1,y_1)=(2,-1)}$

$\boxed{(x_2,y_2)=(-1.69,-2.85)}$



8. Suppose the point (a,b) is on the function from the left. What will the point become after the transformation shown from the function on the right? Indicate all possible answers:

a) $y=|3x-2| \to f(x)=|3x+12|+4$

$f(x)=|3(x+\frac{14}{3})-2|+4$

$\boxed{(a,b)\to(a-\frac{14}{3},\ b+4)}$

b) $y=\frac{4}{3}x+11 \to f(x)=\frac{4x+10}{3}$

Infinite many case:

$\frac{4(x+n)}{3}+\frac{10-4n}{3}=\frac{4x+10}{3}$

$(a,b)\to(a-n,\ b+\frac{10-4n}{3})$; $n>0$, $\frac{10-4n}{3}$

case 1:

$y=\frac{4x+33}{3} \to y=\frac{4(x-\frac{23}{4})+33}{3}$

$(a,b)\to(a-\frac{23}{4},\ b)$

case 2:

$y=\frac{4x+33}{3} \to y=\frac{4x+33}{3}-\frac{23}{3}$

$(a,b)\to(a,\ b-\frac{23}{3})$

c) $y=2^{x}+1 \to f(x)=4(2^{x})-3$

$y=2^{x}+1 \to y=2^{x+2}-3$

$\boxed{(a,b)\to(a-2,\ b-3)}$

d) $y=\frac{1}{x} \to f(x)=-1-x-x^{2}-x^{3}+\ldots\ \{0<x<1\}$

Convergent geometric series:

$r^0+r^1+r^2+r^3+\cdots=\frac{a}{1-r}$

$\{-1<r<1\}$

$y=\frac{1}{x} \to y=\frac{1}{x-1}$

$\boxed{(a,b)\to(a+1,\ b)}$

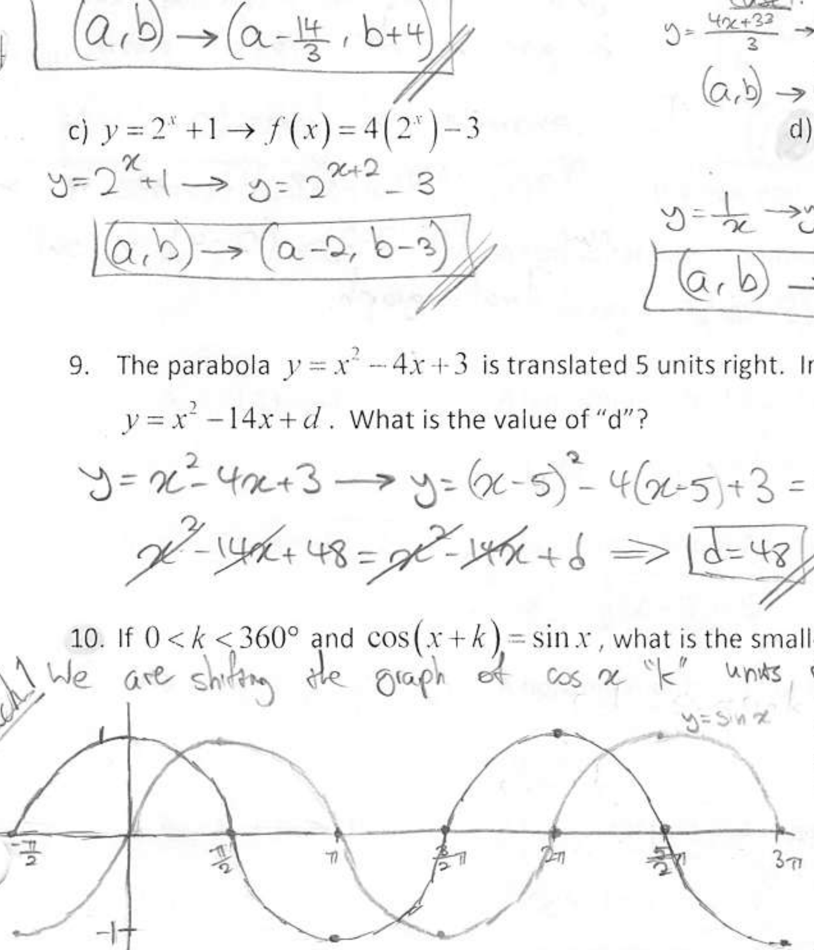
9. The parabola $y=x^2-4x+3$ is translated 5 units right. In this new position, the equation of the parabola is $y=x^2-14x+d$. What is the value of "d"?

$y=x^2-4x+3 \to y=(x-5)^2-4(x-5)+3=x^2-10x+25-4x+20+3=x^2-14x+48$

$\cancel{x^2}-\cancel{14x}+48=\cancel{x^2}-\cancel{14x}+d \Rightarrow \boxed{d=48}$

10. If $0<k<360°$ and $\cos(x+k)=\sin x$, what is the smallest value of "k"?

Approach 1 We are shifting the graph of cos x "k" units to the left.



As you can see, we would have to horizontally shift the graph of cos x $\frac{3\pi}{2}=270°$ to the left in order to make it equal to the graph of sin x.

Smallest value of $\boxed{k=270°}$

Approach 2

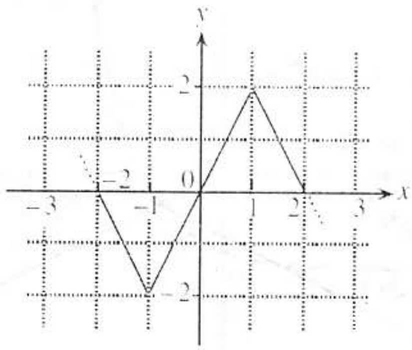
$\cos(a+b)=\cos a\cos b-\sin a\sin b$

$\cos(x+270°)=\cos x\underbrace{\cos 270°}_{=0}-\sin x\underbrace{\sin 270°}_{=-1}$

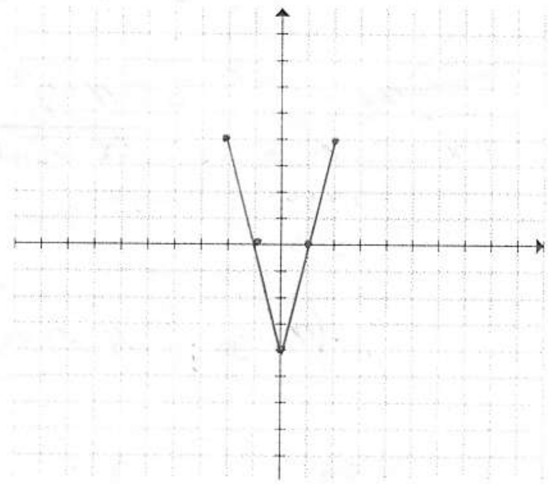
$=\sin x$

$\boxed{k=270°}$

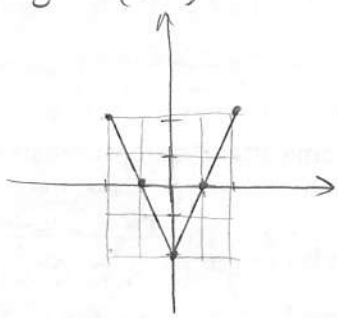
11. The function $f(x)$ has a period of 4. The graph of one period of $y=f(x)$ is shown in the diagram below. Sketch the graph of $y=\left[f(x-1)+f(x+3)\right]$ for $-2 \le x \le 2$.



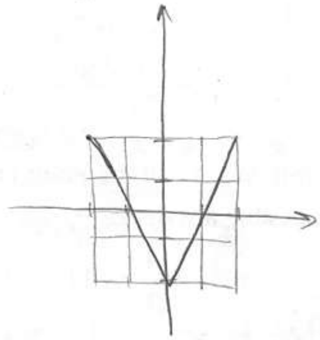
$y = [\,f(x-1)+f(x+3)\,]$



$y = f(x-1)$



$y = f(x+3)$



In order to draw $y=[f(x+3)+f(x-1)]$, we "overlap" the values of the graph. Since $f(x-1)$ and $f(x+3)$ are identical, we end up with $2(f(x-1))$ which just doubles all of our coordinates to give us the graph above.

For example, we have $(0,-2)$ in $f(x-1)$ AND $f(x+3)$, so we will have $(0,-2-2)=(0,-4)$ in our final graph.