Section 3.7 Trigonometric Proofs

Activity: Convert the following angles into sums or differences of 30°, 45°, 60°, 90°, 180°, 270°, or 360°

$75^\circ = 30^\circ \quad\quad ^\circ$

$105^\circ = 60^\circ \quad\quad ^\circ$

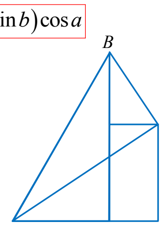
$15^\circ = 45^\circ \quad\quad ^\circ$

$135^\circ = 45^\circ \quad\quad ^\circ$

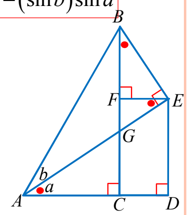
$225^\circ = 270^\circ \quad\quad ^\circ$

I) Prove: $\sin(a+b) = (\sin a)\cos b + (\sin b)\cos a$

$\sin(a+b) =$



II) PROVE: $\cos(a+b) = (\cos a)\cos b - (\sin b)\sin a$



$\cos(a+b) =$

PROVE: $\sin(a-b) = (\sin a)\cos b - (\sin b)\cos a$

PROVE: $\cos(a-b) = (\cos a)\cos b + (\sin a)\sin b$

- The sum and difference identities can be used to find the exact value when you sine/cosine angles that are sums or differences of special angles
- It can only work with angles that are sums or differences of special angles: 30, 60, 90 and 45

EX: FIND EXACT VALUE WITHOUT A CALCULATOR:

$\sin 15^\circ = \sin\left(60^\circ \quad\quad ^\circ\right.,$

Practice: Find Exact value without a calculator:

$a)\ \cos 135^\circ$

$b)\sin\left(\frac{5\pi}{12}\right)$

Prove the following identity: $\cos\left(\frac{\pi}{2}-x\right) = \sin x$

Left Side	*Right Side*

Formula Sheet

$\sin(a+b)=\sin a\cos b+\sin b\cos a$

$\sin(a-b)=\sin a\cos b-\sin b\cos a$

$\cos(a+b)=\cos a\cos b-\sin a\sin b$

$\cos(a-b)=\cos a\cos b+\sin a\sin b$

Practice: Prove $\csc(\pi+x)=-\csc x$

Left Side	*Right Side*

Formula Sheet

$\sin(a+b)=\sin a\cos b+\sin b\cos a$

$\sin(a-b)=\sin a\cos b-\sin b\cos a$

$\cos(a+b)=\cos a\cos b-\sin a\sin b$

$\cos(a-b)=\cos a\cos b+\sin a\sin b$

Practice: Prove $\frac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$

Left Side	*Right Side*

Formula Sheet

$\sin(a+b) = \sin a \cos b + \sin b \cos a$

$\sin(a-b) = \sin a \cos b - \sin b \cos a$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

Prove $3\cos^4 x - 3\sin^4 x = 3\cos 2x$

Left Side	*Right Side*

Prove the Identity $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a(\tan b)}$

Left Side	*Right Side*

III) Double Angle Identities

$\sin(2\theta) = 2\sin\theta \times \cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(2\theta) = 1 - 2\sin^2\theta$

Proving the Double Angle Identities

To prove the Double Angle Identities, use the "Sum Identities"

Ex: Prove $\sin(2\theta) = 2\sin\theta \times \cos\theta$

Left Side	*Right Side*

Ex: Prove $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

Practice: Prove the following Identities

Ex: Prove $\cos(2\theta) = 2\cos^2\theta - 1$

Left Side	*Right Side*

Ex: Prove $\cos(2\theta) = 1 - 2\sin^2\theta$

Proofs using Double Angle Identities

$$\frac{1-\cos 2x}{\sin 2x} = \tan x$$

Left Side	*Right Side*

Practice: Prove the following Identities

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Left Side	*Right Side*

Challenge: Prove $\sin 3x = 3\sin x - 4\sin^3 x$

Left Side	*Right Side*