HOMEWORK DEPOT STUDY SHEETS

Math10 Formula Sheet

This study sheet provides a general review of all topics in the BC Math 10 Principles curriculum. For an extensive online lesson of each topic, visit www.BCMath.ca

Chapter 1: Number Sequences:

1.1 Arithmetic Sequences & Series:

In an arithmetic sequence, each term increases or decreases by adding or subtracting a common difference (d).

$t_n = a + (n-1)d$ — t_n: Value of term of "nth" term.

$S_n = \frac{n}{2}(a+l)$ — S_n The Sum up to the nth term; a: Value of the first term; n: The term number, ie: $1^{st}, 2^{nd}, .$

$S_n = \frac{n}{2}(2a + (n-1)d)$ — d: Common Difference

Ex: Find the 20th term in the sequence:

$5, 13, 21, ...?$ (Note: each term incr. by 8)

$a = 5, d = 8, n = 20 \rightarrow find\ \ t_{20}$

$t_{20} = 5 + (20-1)8 = 157$

The 20th term is 157.

Ex: Find the sum of the sequence:

$(-12)+(-5)+2+9+16+....+107$.

$a = -12,\ d = 7,\ t_n = 107,\ n = ?$

(We don't know how many terms there are)

$t_n = a + (n-1)d$ — 1^{st} *Solve for "n"*

$107 = -12 + (n-1)7 \rightarrow \boxed{n = 18}$

$S_n = \frac{18}{2}(-12+107)$ — 2^{nd} *Solve for "S_{18}"*

$= 855$ — The sum is 855

Ex: If the 4th term in an arithmetic seq. is 7 and the 8th term is 23, find the first term.

$t_1 \quad t_2 \quad t_3 \quad 7 \quad 7+d \quad 7+2d \quad 7+3d \quad 23$

$+d \quad +d \quad +d \quad +d$

8^{th} *term*: $23 = 7 + 4d \rightarrow \boxed{d = 4}$

$t_1 \quad t_2 \quad t_3 \quad 7$ **Go to 1st term by subtracting**

$-d \quad -d \quad -d \quad t_1 = 7 - 4 - 4 - 4 \rightarrow \boxed{t_1 = -5}$

The first term is -5.

1.4: Geometric Sequence:

In a Geometric Sequence, each term increases or decreases by multiplying or dividing by a common ratio (r)

$t_n = a(r)^{n-1}$ — r: Common Ratio

Ex: Find the missing term in the geometric sequence $10, \_\_, \_\_, -270$

$10, \ 10r, \ 10r^2, \ -270$

$\times r \quad \times r \quad \times r$

4^{th} *term*: $-270 = 10r^3$ — Solve for "r"

$-27 = r^3 \rightarrow \boxed{r = -3}$

$10, \ -30, \ 90, \ -270$

$\times(-3) \quad \times(-3) \quad \times(-3)$

The missing terms are -30 and 90.

Ex: A rat colony doubles every week. If a colony has 12 rats now, how many will there be in 20 weeks? (double, $r = 2$)

$a = 12, r = 2, n = 20 \rightarrow find\ \ t_{20}$

$t_{20} = 12(2)^{20-1} = 12(2)^{19} = 6291456\ rats$

1.5 Exponent Laws:

$x^m \bullet x^n = x^{m+n}$ — ie: $(3^5)(3^2) = 3^7$

$x^m \div x^n = x^{m-n}$ — ie: $(3^5) \div (3^2) = 3^3$

$(x^m)^n = x^{m \bullet n}$ — ie: $(3^2)^5 = 3^{10}$

$(x^m y^n)^k = x^{m \bullet k} y^{n \bullet k}$ — ie: $(3^2 \bullet 4^3)^5 = (3^{10} 4^{15})$

$\left(\frac{x^m}{y^n}\right)^k = \left(\frac{x^{m \bullet k}}{y^{n \bullet k}}\right)$ — ie: $\left(\frac{3^2}{4^3}\right)^5 = \left(\frac{3^{10}}{4^{15}}\right)$

$x^0 = 1$ $\quad$ $x^{-n} = \frac{1}{x^n}$ — ie: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Ex: Simplify

i $\frac{(a^3b^2)^3}{(a^{-1}b^3)^{-2}} \rightarrow \frac{(a^3b^2)^3}{(a^{-1}b^3)^{-2}} = \frac{a^9b^6}{a^2b^{-6}} = a^7b^{12}$

ii) $(-a^3)^3 = (-1)^3(a^3)^3 = -1(a^9) = -a^9$

1.7 Sequences in Tables:

$I = P \times r \times t$

I: Interest Earned $\quad$ r: Interest Rate-decimal form

t: Time (years) $\quad$ P: Principle-beginning amount

Ex: $1000 was invested at 3% for 3 years. How much interest is earned?

Yr	Open Bal.	Int. (r)	Int. Earned	Close Bal.
1	$1000	0.03	$30	1030
2	1030	0.03	30^{.90}$	1060$^{.90}$
3	1060$^{.90}$	0.03	31^{.83}$	1092$^{.73}$
4	1092$^{.73}$	0.03	32^{.78}$	1125$^{.51}$
5	1125$^{.51}$	0.03	33^{.77}$	1159$^{.28}$

Total Interest Earned = $159.28

Closing Balance= Open. Balance + Int. Earned

Recursive: A sequence where every term requires the previous term.

1.6/1.7 Date in Tables:

$PST\ Rate = \frac{PST\ Cost}{Object\ Cost}$

Appreciation: Increase in value of an object by a percentage.

Depreciation: decrease in value of an object by a percentage.

Ex: A $45000 minivan depreciates by 15%. What is the worth after 3 years?

Yr	Open Cost	Deprec. 15 %	Close Cost
1	45,000	6750	38250
2	38,250	5737	32512$^{.50}$
3	32512	4876$^{.88}$	27635$^{.63}$

Chapter 2: Real Numbers:

2.1: Radicals:

Radicals are numbers that have $\sqrt{\ }$ sign. When simplifying radicals, easier to use perfect squares and perfect cubes

n	1	2	3	4	5	6	7	8
n^2	1	4	9	16	25	36	49	64
n^3	1	8	27	64	125	216	343	512

$\sqrt{x}$ means square root $\quad$ $\sqrt[3]{x}$ means cube root

Simplifying Radicals Steps:

1st: Split the number into multiples of different perfect squares

2nd Square root each perfect square separately

3rd Numbers that are not perfect squares stay in the root sign

Ex: Simplify each of the following:

$i)\sqrt{32} \rightarrow \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

$ii)\sqrt{72} \rightarrow \sqrt{9} \times \sqrt{4} \times \sqrt{2} = 3 \times 2 \times \sqrt{2} = 6\sqrt{2}$

$iii)\sqrt[3]{192} \rightarrow \sqrt[3]{64} \times \sqrt[3]{3} = 4\sqrt[3]{3}$

$iv)\sqrt[3]{-81} \rightarrow \sqrt[3]{-1} \times \sqrt[3]{27} \times \sqrt[3]{3} = -1(3)\sqrt[3]{3} = -3\sqrt[3]{3}$

2.2 Exponent Laws with Radicals

The exponents are fractions. The numerator becomes the power and the denominator is the root.

$\sqrt[n]{x} = x^{\frac{1}{n}}$ $\qquad$ $A^{\frac{1}{3}} = \sqrt[3]{A}$

$x^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{x}}$ $\qquad$ $B^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{B}}$

$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{(x^m)}$ $\qquad$ $C^{\frac{2}{3}} = \left(\sqrt[3]{C}\right)^2$

$x^{-\frac{m}{n}} = \frac{1}{\left(\sqrt[n]{x}\right)^m}$ $\qquad$ $D^{-\frac{2}{3}} = \frac{1}{\left(\sqrt[3]{D}\right)^2}$

Note: If the exponent is negative, flip the base (reciprocal)& change the exponent to a positive

Ex: Simplify:

$(-32)^{\frac{2}{5}} = \left(\sqrt[5]{-32}\right)^2 = (-2)^2 = 4$

$\left(\frac{25}{49}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{25}{49}}\right)^3 = \left(\frac{5}{7}\right)^3 = \frac{125}{343}$

$\left(\frac{-27}{64}\right)^{-\frac{2}{3}} = \left(\frac{64}{-27}\right)^{\frac{2}{3}} = \left(\frac{\sqrt[3]{64}}{\sqrt[3]{-27}}\right)^2 = \left(\frac{4}{-3}\right)^2 = \frac{16}{9}$

$\left(\sqrt{x^4}\right)\left(\sqrt{x^2}\right) = \left(x^{\frac{4}{2}}\right)\left(x^{\frac{2}{2}}\right) = x^{2+1} = x^3$

$(-x)^{\frac{2}{3}} = \left(\sqrt[3]{-x}\right)^2 = \left(\sqrt[3]{-1} \times \sqrt[3]{x}\right)^2 = \left(-1 \times \sqrt[3]{x}\right)^2 = \left(\sqrt[3]{x}\right)^2$

$\frac{1}{\sqrt[4]{x^3}} = \frac{1}{x^{\frac{3}{4}}} = x^{-\frac{3}{4}}$

2.5 Irrational Numbers:

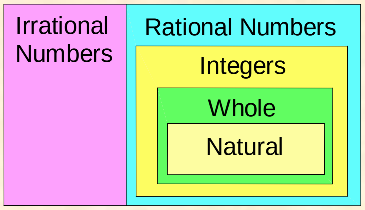
Irrational Numbers: Decimal values don't stop, and no pattern. Usually the square root of a non-perfect square.

Ie: 1.717117111…., $\pi, \sqrt{3}$

Rational Numbers: Decimal values stop or repeats with a pattern. Can be written as a fraction with two integers.

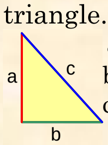
Ie $\frac{2}{3}$, 1.1111..., $3.\overline{33}, \sqrt{16}$

Integers: -3,-2,-1,0,1,2,3,4..
Whole Numbers: 0,1,2,3,4,5
Natural Numbers: 1,2,3,4,5



Pythagorean Theorem: $a^2+b^2=c^2$

The Pythagorean Theorem is for finding missing sides in a right triangle.



a-height
b-base
c-hypotenuse

Note: Need at least 2 sides to find the third side.

Ex: Given hyp=15, base=10, find height

$\rightarrow a=x, b=10, c=15$ — Find a, b, c

$a^2=x^2, b^2=100, c^2=225$ — Find a^2, b^2, c^2

$x^2+100=225$ — Solve for "x"

$x^2=125$

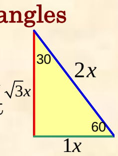
$x=\sqrt{125}=5\sqrt{5}$ — Simplify radical

Pythagorean Triples: $a^2+b^2=c^2$

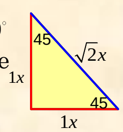
3,4,5 5,12,13 8,15,17, 7,24,25 20,21,29
9,40,41 6,8,10 10,24,26 16,30,34 14,28,50

Special Triangles

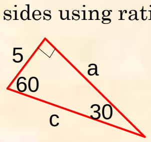
$30^\circ-60^\circ-90^\circ$
Equilateral triangle cut in half



$45^\circ-45^\circ-90^\circ$
Right angle isosceles triangle



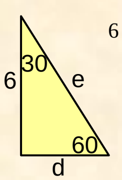
Ex: Find the length of the missing side:
The triangles are similar AAA, so compare the sides using ratios.



$5 \sim x$

$a \sim x\sqrt{3} \rightarrow a=5\times\sqrt{3}=5\sqrt{3}$

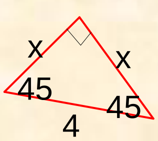
$c \sim 2x \rightarrow c=5\times 2=10$



$6 \sim x\sqrt{3} \quad x\sqrt{3}=6 \rightarrow x=\frac{6}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} \rightarrow \boxed{x=2\sqrt{3}}$

$e \sim 2x \quad e=2x=2\left(2\sqrt{3}\right)=\boxed{4\sqrt{3}}$

$d \sim x \quad d=x \rightarrow \boxed{d=2\sqrt{3}}$



Rationalize the denominator!

$4=x\sqrt{2} \rightarrow x=\frac{4}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}=2\sqrt{2}$

so $\boxed{x=2\sqrt{2}}$

2.7/2.8 Multiplying& Dividing Radicals

When multiplying radicals, first multiply the number outside of the radicals. Then multiply the number inside the radicals and simplify.

ie: i) $4\sqrt{3}\times 2\sqrt{5}=8\sqrt{15}$ ii) $\frac{8\sqrt{15}}{2\sqrt{3}}=4\sqrt{5}$

Ex: Simplify $3\sqrt{2}\left(4\sqrt{6}-2\sqrt{8}+2\sqrt{24}\right)$

$3\times4\sqrt{2}\sqrt{6}-3\times2\sqrt{2}\sqrt{8}+3\times2\sqrt{2}\sqrt{24}$

$12\sqrt{12}-6\sqrt{16}+6\sqrt{48}$

$12\sqrt{4}\times\sqrt{3}-6(4)+6\sqrt{16}\times\sqrt{3}$

$24\sqrt{3}-24+24\sqrt{3}\rightarrow\boxed{48\sqrt{3}-24}$

Ex: Expand

$\sqrt{6}\left(\sqrt{7}+\sqrt{8}\right)=\sqrt{42}+\sqrt{48}=\sqrt{42}+4\sqrt{3}$

Ex: FOIL:

$\left(3-\sqrt{5}\right)^2=\left(3-\sqrt{5}\right)\left(3-\sqrt{5}\right)$

$=9-3\sqrt{5}-3\sqrt{5}+5=14-6\sqrt{5}$

2.9: Adding & Subtracting Radicals:

When add/subtr radicals, only the number outside changes. The number in the radical does not change.

You can only add/subtract radicals if they are "LikeTerms" (radicals are the SAME). If radicals are not liketerms you CANNOT add/subtract them.

Ex Simplify:

$5\sqrt{3}+2\sqrt{3}-\sqrt{3}=6\sqrt{3}$

$5\sqrt{3}+3\sqrt{5}=$ (Not Liketerms)

2.10. Rationalizing Radicals

Never leave radicals in the denominator. Rationalize the denominator by multiply -ing the top&bottom by the same radical.

Ex: Rationalize:

$\frac{1}{\sqrt{2}}\rightarrow\frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\frac{6}{\sqrt{3}}\rightarrow\frac{6}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}}{3}=2\sqrt{3}$

If the denominator is a binomial with a radical, rationalize by multiplying top and bottom by the conjugate. (change the sign in between)

ie: $4+\sqrt{5}\rightarrow4-\sqrt{5}$

$5-3\sqrt{2}\rightarrow5+3\sqrt{2}$

Ex: Rationalize:

i) $\frac{\left(6+\sqrt{5}\right)}{\left(3-\sqrt{5}\right)}\rightarrow\frac{\left(6+\sqrt{5}\right)}{\left(3-\sqrt{5}\right)}\frac{\left(3+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)}=\frac{23+9\sqrt{5}}{4}$

ii) $\frac{(16)}{3-\sqrt{5}}\rightarrow\frac{(16)}{3-\sqrt{5}}\frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{48+16\sqrt{5}}{9-5}$

$=\frac{48+16\sqrt{5}}{4}=12+4\sqrt{5}$

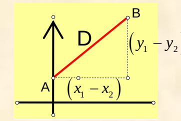
After rationalizing, the radical in the denominator will be gone.

Chapter 3: Line Segments:

3.1: Distance between two points

$\boxed{D=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}}$

ie: Pythagorus



3.2 Midpoint of a Line:

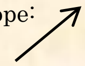
$\boxed{\left(x_m,y_m\right)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)}$

Average of the x coordinates & y coordinates

3.3 Slope of a Line

$\boxed{slope=\frac{rise}{run}=\frac{y_1-y_2}{x_1-x_2}=\frac{\Delta y}{\Delta x}}$

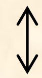
Positive slope: Slant UP



Negative Slope: Slant DOWN

Horizontal Line: Slope = 0

Vertical Line: Slope is undefined



Ex: Find the distance, midpoint, & slope between the A $(-3,4)$, B $(4,-1)$

$D=\sqrt{\left(-3-4\right)^2+\left(4-(-1)\right)^2}=\sqrt{(-7)^2+(5)^2}=\boxed{\sqrt{74}}$

$M:\left(x_m,y_m\right)=\left(\frac{-3+4}{2},\frac{4+(-1)}{2}\right)=\boxed{\left(\frac{1}{2},\frac{3}{2}\right)}$

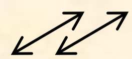
$Slope: m=\frac{4-(-1)}{-3-4}=\boxed{\frac{5}{-7}}$

Ex: Given $A(3,-5)$ $B(6,k)$ **and the slope is** $\frac{1}{2}$. **Find the value of "k"**

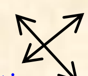
$\frac{1}{2}=\frac{k-(-5)}{6-3}\rightarrow\frac{1}{2}=\frac{k+5}{3}\rightarrow3=2k+10\rightarrow\boxed{\frac{-7}{2}=k}$

3.4 & 3.5 Slopes of Parallel Lines & Perpendicular Lines:

If two lines are Parallel → Slopes are equal. Slanted in the same angle



If two lines are Perpendicular $(\perp)$, intersect at 90° →Slopes are Negative Reciprocals



(flip& change sign) $\boxed{m\rightarrow-\frac{1}{m}}$

Note: The product of two neg. recip. is -1.

Ex: Find the neg. recip. of each number

$i)\frac{2}{3}\rightarrow-\frac{3}{2}$ $\quad ii)\frac{18}{3}\rightarrow-\frac{1}{6}$ $\quad iii)-2\rightarrow\frac{1}{2}$

Ex: Is VH $\perp$ DY? $V(1,-2)H(5,4)D(6,4)Y(12,0)$

$m_{VH}=\frac{4-(-2)}{5-1}=\frac{6}{4}=\frac{3}{2}$ ↘ *Slopes* are neg. reciprocals

$m_{DY}=\frac{0-4}{12-6}=\frac{-4}{6}=-\frac{2}{3}$ ↗ *Lines* are perpendicular

Ex: Given each slope, indicate T/F:

$AB=\frac{5}{6}$ $\quad CD=\frac{10}{12}$ $\quad DE=\frac{-15}{18}$ $\quad FG=\frac{30}{-36}$ $\quad HI=\frac{-12}{10}$

a) AB & CD are parallel - True
b) CD & DE are parallel - False
c) DE & FG are parallel - True
d) HI & AB are perpendicular - True

Chapter 4: Straight Lines

Equation of a Line Property:

The coordinate of every point on a line will satisfy its line equation.

4.2 Equation of a Straight Line

2 things you will always need to find the equation of a line: Slope and y-intercept

$\boxed{y = mx + b}$ $\quad m: slope \quad b:$ y-intercept

$i)\, y = -3x + 11 \quad m = -3 \quad b = 11$

$ii)\, y = 12 - 4x \quad m = -4 \quad b = 12$

$iii)\, y = \dfrac{5x-4}{3} \quad m = \dfrac{5}{3} \quad b = \dfrac{-4}{3}$

Ex: Given the slope & a point, find equation of the line: $m = \frac{1}{2}$ point $(4,3)$

1st $m = \frac{1}{2} \rightarrow y = \frac{1}{2}x + b$

2nd Find b: $3 = \frac{1}{2}(4) + b \rightarrow 1 = b$

The line equation is $\boxed{y = \frac{1}{2}x + 1}$

Ex: Given the two points, find the equation of the line: $(3,2) \& (-1,8)$

1st $m = \dfrac{8-2}{-1-3} = \dfrac{6}{-4} = \dfrac{3}{2} \rightarrow y = \frac{3}{2}x + b$

2nd Find b: $8 = \frac{3}{2}(-1) + b \rightarrow b = \frac{19}{2}$

The line equation is $\boxed{y = \frac{3}{2}x + \frac{19}{2}}$

Note: At the Y-intercepts, the x co-ordinate is zero. Likewise, at the X-intercept, the y-coordinate is zero.

Ex: Find the x & y intercepts: $y = 4x + 12$

$y\text{int}: (x=0) \rightarrow y = 4(0) + 12 \rightarrow \boxed{y = 12}$

$x\text{int}: (y=0) \rightarrow 0 = 4x + 12 \rightarrow \boxed{x = -3}$

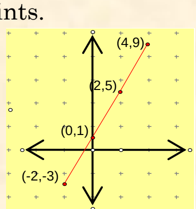
4.1) How to Graph a Line: $\boxed{y = mx + b}$

1st Method:

Make a table of values. Pick values for x, and then use the equation to solve for the y-values. Need atleast 2 points.

Ex: Graph $y = 2x + 1$

x	-2	0	2	4
y	-3	1	5	9

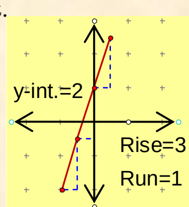


2nd Method:

Find the slope and y-intercept. Draw the first point on the y-inter. and apply the slope to get the next point.

Ex: Graph $y = 3x + 2$

$\rightarrow m = 3, b = 2$



3rd Method: Find both the 'x' & 'y' intercept. Connect the two intercepts.

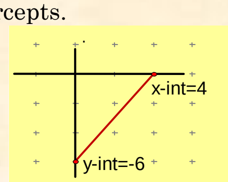
Ex: Graph $y = \frac{3}{2}x - 6$

$\rightarrow y - \text{int} = -6$

$\rightarrow x - \text{int} = 4$

make $y = 0$,

solve for x



4.5: Line Equation: $Ax + By + C = 0$

This is a line equation in standard form. To find the slope and y intercept, isolate the y variable.

$\boxed{m = \frac{-A}{B}}$, $\boxed{y - \text{intercept} = \frac{-C}{B}}$

Ex: Find slope and y-int. $2x + 3y - 5 = 0$

$2x + 3y - 5 = 0$

$3y = -2x + 5$

$y = \dfrac{-2x}{3} + \dfrac{5}{3} \rightarrow m = \dfrac{-2}{3}, b = \dfrac{5}{3}$

Constant Slope Property

Despite which two points on a line are chosen to calculate the slope, the slope will always be the same.

Ex: Given that points $(2,-1)(5,11)(8,k)$ **are all one the same line, find "k".**

$\dfrac{-1-11}{2-5} = \dfrac{11-k}{5-8} \rightarrow \dfrac{-12}{-3} = \dfrac{11-k}{-3}$

$36 = -3(11-k)$ Cross Multiply

$-12 = 11 - k$

$k = 23$

Ex: Given the line $y = \dfrac{-2}{3}x + 3$ **find a perpendicular line that crosses (3,2).**

1st Slope of perpendicular line: $m = \dfrac{3}{2}$

2nd Line equation: $y = \dfrac{3}{2}x + b \rightarrow \quad 2 = \dfrac{3}{2}(3) + b$

$2 = 4.5 + b \rightarrow \quad b = -2.5$

$\rightarrow \boxed{y = \dfrac{3}{2}x - 2.5}$

Applications of Linear Functions

Ex: The cost of ordering "n" textbooks is given by the formula: $C = 100 + 80n$

a)make a table of values up to 100 books

b) Graph c)How many books can be purchased with $4000?

1st base cost = $100; cost per book =$80

a)

n	0	20	40	60	80	100
C	100	1700	3300	4900	6500	8100

c)

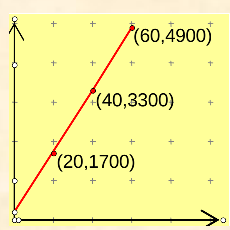
$4000 = 100 + 80n$

$3900 = 80n$

$48.75 = n$

$\boxed{48 \text{ books}}$

b)



Ex: Rogers charges $35/month at 0.20 per minute. Bell charges $25/month at 0.25 per minute. After how many minutes will Bell exceed Rogers in cost?

$35 + 0.20n < 25 + 0.25n$

$10 < 0.05n$

$200 < n$

After 200 minutes Bell will exceed Rogers in cost.

Chapter 5: Functions

5.1: What is a Function?

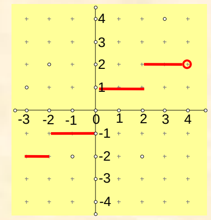
A rule/equation where the input variable (x) will create a output variable (y).

Domain: all possible X values (Input variable, Independent)

Range: all possible Y values (Output variable, Dependent)

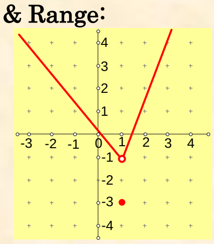
When interpreting graphs, take note of what variables are on the x and y axis. Sometimes it's "Cost (y) vs Time (x)", "Speed(y) vs Time (x)", or "Distance(y) vs. Time(x)"

Ex: Indicate the Domain & Range:



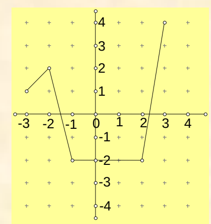
D: $-3 \le x < 4$

R: $y = -2, -1, 1, 2$



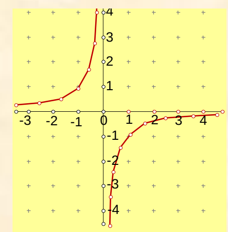
D: $x \in \mathbb{R}$

R: $y < -1;\ y = -3$



D: $-3 \le x \le 3$

R: $-2 \le y \le 4$



D: $x \in \mathbb{R}, x \ne 0$

R: $y \in \mathbb{R}, y \ne 0$

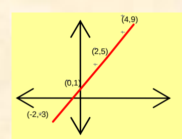
5.3 Different types of Function:

Linear Functions

(Straight lines)

$y = mx + b$

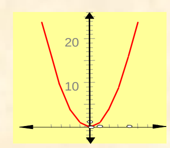
$m: Slope \quad b: y - \text{intercept}$



Quadratic Functions

$y = x^2$

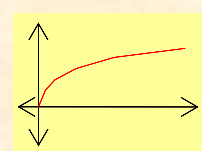
(Parabolas)



Root Functions

$y = \sqrt{x};\quad$ D: $x > 0$

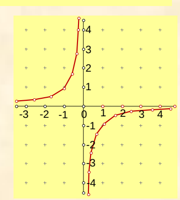
(Square root)



Reciprocal Functions

$y = \dfrac{1}{x}, \quad x \ne 0$

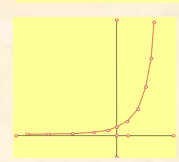
("x" in denominator)



Exponent Functions

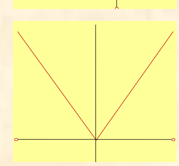
$y = A^x, \ A > 0$

("x" is a power)



Absolute Functions

$y = |x|$



5.6: Function Notation:

General Rule: Whatever 'x' becomes inside the brackets, 'x' will become the same thing in the equation.

Ex: $f(x)=3x+4, \quad g(x)=\sqrt{x-3}$ Find:

i) $f(x+1)=3(x+1)+4$

ii) $f\left(\frac{1}{x}\right)=3\left(\frac{1}{x}\right)+4$

iii) $g(f(x))=\sqrt{f(x)-3}=\sqrt{(3x+4)-3}$
$=\sqrt{3x+1}$

iv) $f(g(12))=3(g(12))+4=3\left(\sqrt{12-3}\right)+4$
$=3\left(\sqrt{9}\right)+4=3(3)+4=13$

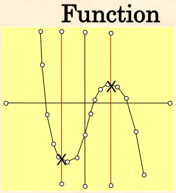
5.7: Relations vs Functions:

Functions are one to one, where one input value yields only one output value.

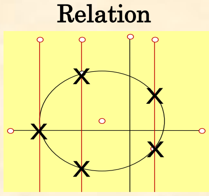
Relations can be one to one, one to two, or one to three and so on. One input vale can yield more than one output value.

Vertical Line Test: Any vertical line can cross only one point at a time.

Function



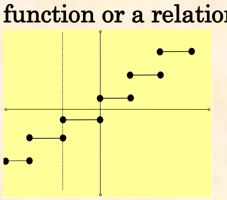
Relation



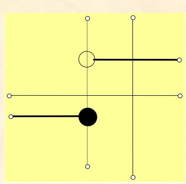
Relations
Functions

All functions are relations BUT Not all relations are functions

Ex: Indicate if each of the following is a function or a relation?



Not a function



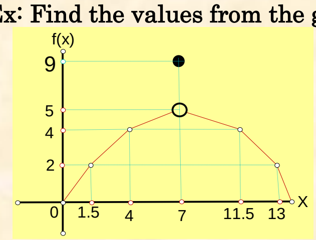
Function

Left graph not a function because vertical line crossed two solid dots: not one to one.

Right graph is a function because one dot is hollow: does not have a value.

Note: A hollow dot means a value does not exist at the point. A solid dot means that a value does exist.

Ex: Find the values from the graph:



$f(1.5)=2 \qquad f(7)=9 \qquad f(4)=4 \qquad f(11.5)=4$

Chapter 6: Polynomials

There are 3 types of factoring: GCF, Trinomials and Difference of Squares

Note: GCF: Greatest Common Factor
$ie: GCF\langle 18,12\rangle=6 \quad GCF\langle 21,49\rangle=7$

6.4: Factoring Out Common Factors:

First look for the GCF of all terms, then divide all terms by the GCF.

Ex: Factor $9x^3y^5+12x^6y^2$

$GCF=3x^3y^2 \rightarrow 3x^3y^2\left(3y^3+4x^3\right)$

6.6: Factoring Trinomials $x^2+Bx+C=0$

Steps:
1. Find two numbers that mult. to C
2. Pick the Pair that Adds to B
3. Use the two numbers for the two binomials

Ex: Factor: $\left(x^2+11x+24\right)$

$3\times 8=24, \quad 3+8=11 \rightarrow (x+8)(x+3)$

6.7: Factoring Trinomials $Ax^2+Bx+C=0$

When factoring trinomials with $A\neq 1$, there are 3 methods of factoring.

1st Method: Grouping Method:

i Split B as the sum of two numbers where each are factors/multiples of A or C
ii Group A with one factor& C with the other
iii Factor out common factor, the binomial should be the same, then factor the binomial

Ex: Factor $6x^2+7x+2$

$6x^2+3x+4x+2 \qquad (7x=3x+4x)$

$\left(6x^2+3x\right)+(4x+2) \qquad$ group A &C with factors

$3x(2x+1)+2(2x+1) \qquad$ factor out common factor

$(2x+1)(3x+2) \qquad$ factor out the binomial

2nd Method: Criss-Cross Method

i Find 2 factors of "A" and "C"
ii Multiply factors, one each from A&C
iii The sum of the 2 products should equal B

Note: there will be many combinations, pick the one that adds to B.

iv Factors of A placed first& factors of C placed second in each binomial. Factors that are multiplied can not be in same binomial

$Ex: Factor\ 6x^2+7x+2$

$3 \leftrightarrow 2=6 \searrow$ adds to 8 $\qquad 3 \leftrightarrow 1=3 \searrow$ adds to 7!
$2 \leftrightarrow 1=2 \nearrow \qquad\qquad\quad 2 \leftrightarrow 2=4 \nearrow$

The sum needs to be equal to B (7), so second one works. Place factors of "6" first in each binomial, then factors of "2" *afterwards*. $3\ \&\ 1$ must be in different brackets.

$\rightarrow \ (3x+2)(2x+1)$

3rd Method: Bum Method."

i) Multiply C with A → A = 1.
ii) Factor it into 2 binomials
iii) Bring A back in both binomials front of x's
iv) Eliminate any common factors in binomials

Ex: Factor $6x^2+7x+2$

$\rightarrow x^2+7x+12=(x+3)(x+4) \qquad$ Mult. 6 with 2

$\rightarrow (6x+3)(6x+4) \qquad$ Bring the 6 back

$\rightarrow \left(\frac{6x+3}{3}\right)\left(\frac{6x+4}{2}\right) \qquad$ Eliminate (Bum) out any common factors

$\rightarrow \boxed{(2x+1)(3x+2)}$

6.8 Factoring a Difference of Squares

This is used when you have a perfect square subtracting another perfect square.

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

Ex:Factor: $25x^2-64y^2$

$\rightarrow a^2=25x^2, b^2=64y^2 \rightarrow a=5x, b=8y$

$=(5x+8y)(5x-8y)$

ii) $(x+1)^2-9$

$\rightarrow a^2=(x+1)^2, b^2=9 \rightarrow a=(x+1), b=3$

$=\left[(x+1)+3\right]\left[(x+1)-3\right]$

6.10 Long Division with Polynomials

$$\frac{Quotient(Q)}{Divisor(P)\overline{)Dividend(D)}} \qquad Remainder(R)$$

Ex#1) Divide $3x^3+11x^2-6x-10$ by $x+4$

$3x^2 \quad -x \quad -2$
$x+4\overline{)3x^3+11x^2-6x-10} \qquad$ *Divide $3x^3$ by $x \rightarrow 3x^2$ in Q*
$-\left(3x^3+12x^2\right) \qquad$ *Mult. $3x^2$ by divisor → subtract*
Carry down −6x
$-x^2-6x \qquad$ *Divide $-x^2$ by $x \rightarrow -x$ in Q*
$-\left(-x^2-4x\right) \qquad$ *Mult. − x by divisor → subtract*
Carry down −10
$-2x-10 \qquad$ *Divide −2x by x → −2 in Q*
$-(-2x-8) \qquad$ *Mult. − 2bydivisor → subtract*
$R=-2$

Division Statement $\boxed{D=PQ+R}$

$3x^3+11x^2-6x-10=\left(3x^2-x-2\right)(x+4)-2$

6.10 Synthetic Division:

1.Use the divisor to find number on the left
2.Bring the first number down, you add downwards
3. Multiple each number on the bottom with divisor to find next number diagonally

Ex#1) Divide $3x^3+11x^2-6x-10$ by $x+4$

$-4 \,|\, 3 \quad 11 \quad -6 \quad -10 \qquad \rightarrow Quot: 3x^2-1x-2$
$\quad\ \downarrow \quad -12 \quad 4 \quad 8$
$\quad\ 3 \quad -1 \quad -2 \quad -2 \qquad$ Remainder: -2

Note: Degree of terms in dividend must in descending order.

Two Reminders with Synthetic Division

i)If the Dividend is missing a term (skip in the exponents), replace missing term with coefficient of zero

ie: $3x^3-2x+1 \rightarrow 3x^3+0x^2-2x+1$ (skip from x^3 to x^1)

2. If Divisor has a coeff. for the x-term, solve for "x" from the divisor. Do synthetic division. At the end, factor out the same coefficient from the quotient.

Ex: Divide: $4x^3+6x^2-2x+4 \quad \div \quad 2x+1$

$2x+1 \rightarrow x=-\frac{1}{2} \qquad Dividend=\left(x+\frac{1}{2}\right)\left(4x^2+4x-4\right)+6$

$-\frac{1}{2} \,|\, 4 \quad 6 \quad -2 \quad 4 \quad \rightarrow \qquad$ D $=2\left(x+\frac{1}{2}\right)\left(2x^2+2x-2\right)+6$
$\qquad\quad -2 \quad -2 \quad 2 \qquad\qquad$ D $=(2x+1)\left(2x^2+2x-2\right)+6$
$\quad 4 \quad 4 \quad -4 \quad 6$

Quotient (Q): $2x^2+2x-2$ Divisor (P): $2x+1$

Dividend $4x^3+6x^2-2x+4$ Remainder $R=6$

Division Statement:

$4x^3+6x^2-2x+4=\left(2x^2+2x-2\right)(2x+1)+6$

Formulas for Volumes:

Sphere $V=\frac{4}{3}\pi r^3$ Cube $V=S^3$

Cylinder $V=\pi r^2\times h$

Rectangular Prism: $V=l\times w\times h$

Formulas for Surface Area:

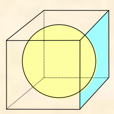
Sphere $SA=4\pi r^2$ Cube $SA=6s^2$

Cylinder: $SA=2(\pi r^2)+(2\pi r)h$

Rectangular Prism: $S=2(lw+lh+wh)$

Applications of Polynomials:

Ex: A cube with 10cm sides has a sphere inside. Find the volume of the largest sphere. Find the ratio of the sphere's surface area compared to the cube.



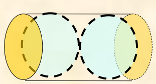
$r=5cm\to V=\frac{4}{3}\pi(5)^3=523.3cm^3$

$SA_{Sphere}=4\pi(5)^2=314cm^2$

$SA_{Cube}=6(10)^2=600cm^2$

$\%=\frac{314}{600}=52.3\%$

Ex: 2 spheres with radius of 5cm are placed in a cylinder. Find the % of the sphere's volume compared to the cylinder.

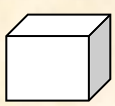


$2V_{sphere}=2\left(\frac{4}{3}\pi(5)^3\right)=1046.7cm^3$

$V_{cylinder}=\left(\pi(5)^2\right)10=1570cm^3$

$\%=\frac{1046.7}{1570}=66.7\%$

Ex: A box with a volume of 200cm³, is increased by a scale factor of 3. Find the new volume.



$V_{original}=l\times w\times h$

$V_{new}=(3l)\times(3w)\times(3h)$

$=27(l\times w\times h)$

$=27(V_{original})\to 27$times larger

Ch7: Rational Expressions (RE)

NPV(Non-Permissible Values) Values of variables that make the expression undefined, which means making the denominator equal to zero.

Ex: Find all NPV's (Denominators only)

$\frac{2x-3}{3x}$	$\frac{x^2-3x+1}{x^2-81}$	$\frac{2x+4}{x^2+5x+4}$
$3x=0$	$x^2-81=0$	$x^2+5x+4=0$
$x=0$	$(x+9)(x-9)=0$	$(x+1)(x+4)=0$
	$x=\pm 9$	$x=-1,-4$
$NPV: x\neq 0$	$NPV: x\neq \pm 9$	$NPV: x\neq -1,-4$

Some expressions may not have an NPV:

$\frac{2x^2-3x}{5}\to$ no variable in denominator

$\frac{3x-5x^2}{x^2+1}\to x^2+1\neq 0$

7.3 Multiplying & Dividing RE:

When multiplying fractions, simplify by cancelling out common factors in the numerator and denominator. When dividing fraction, flip the second fraction first, and then simplify.

Ex: Simplify:

$\frac{5}{24}\times\frac{8}{15}$ $\qquad$ $\frac{10}{21}\div\frac{30}{49}$

$\frac{1}{3}\times\frac{1}{3}$ $\qquad$ $\frac{10}{21}\times\frac{49}{30}$

$\frac{1}{9}$ $\qquad$ $\frac{1}{3}\times\frac{7}{3}=\frac{7}{9}$

Ex: Simplify and find all NPV's

i) $\frac{4x+8}{2x+4}$ $\qquad$ ii) $\frac{(x-3)(x+2)}{(x-4)(x+1)}\times\frac{(x-4)}{(x-3)}$

$=\frac{4(x+2)}{2(x+2)}=2$ $\qquad$ $=\frac{(x+2)}{(x+1)}$

NPV: $x\neq -2$ $\qquad$ $NPV: x\neq 4,-1,3$

When looking for the NPV, consider the denominator for all steps!

NOTE: $\frac{(x-5)}{(5-x)}=-1$ $\qquad$ $\frac{(x-5)}{(x-5)}=1$

7.4/7.5: Adding & Subtracting RE

You can only add or subtract rational expressions if they have a common denominator. Find the LCD.

Ex: Simplify by Adding or Subtracting:

$ie: \frac{4}{3x}+\frac{7x}{6}$ $\qquad$ $ie: \frac{4}{x+1}-\frac{3}{x}$

$\frac{8}{6x}+\frac{7x^2}{6x}$ $\qquad$ $\frac{4}{(x+1)}\frac{(x)}{(x)}-\frac{3}{x}\frac{(x+1)}{(x+1)}$

$\frac{8+7x^2}{6x}$ $\qquad$ $\frac{4x-3(x+1)}{x(x+1)}$

$=\frac{4x-3x-3}{x(x+1)}\to \quad =\frac{x-3}{x(x+1)}$

Ex: Simplify:

$\frac{(x+3)}{x^2+11x+24}-\frac{2x+10}{x^2+11x+30}$ Factor

$\frac{(x+3)}{(x+3)(x+8)}-\frac{2(x+5)}{(x+5)(x+6)}$ Simplify

$\frac{1}{(x+8)}-\frac{2}{(x+6)}$ LCD: $(x+8)(x+6)$

$\frac{1}{(x+8)}\frac{(x+6)}{(x+6)}-\frac{2}{(x+6)}\frac{(x+8)}{(x+8)}$

$\frac{1(x+6)-2(x+8)}{(x+8)(x+6)}$

$\frac{-x-2}{(x+8)(x+6)}\to NPV: x\neq -8,-6$

7.6 Solving Rational Expressions (RE):

Solving means finding a value for x where the equation is equal on both sides. You can "solve" only when the equation has an "equal" sign. Solve RE by eliminating the denominator with the LCD.
Solving is different from simplifying. You simplify when there isn't an equal sign.

When there are two terms only, cross multiply them.

Ex: Solve for "x" and all NPV's

$ie: \frac{4}{3x}+\frac{7x}{6}$ $\qquad$ $ie: \frac{4}{x+1}-\frac{3}{x}$

$\frac{8}{6x}+\frac{7x^2}{6x}$ $\qquad$ $\frac{4}{(x+1)}\frac{(x)}{(x)}-\frac{3}{x}\frac{(x+1)}{(x+1)}$

$\frac{8+7x^2}{6x}$ $\qquad$ $\frac{4x-3(x+1)}{x(x+1)}$

$=\frac{4x-3x-3}{x(x+1)}\to \quad =\frac{x-3}{x(x+1)}$

When Solving RE with three terms, find the LCD. Multiply all terms with the LCD to cancel out the denominator in each term. Then solve algebraically.

Ex: Solve for "x" and all NPV's

$x+\frac{8}{x+5}=4$ move terms with LCD together

$\frac{8}{x+5}=4-x$ cross multiply

$8=(4-x)(x+5)$ FOIL

$8=4x+20-x^2-5x$ Move Left

$x^2+x-12=0$ Factor

$(x-3)(x+4)=0$

$x=3,-4$

Extraneous Roots: A solution that is also an NPV. Extraneous Roots are rejected.

Ex: Solve for "x" and all NPV's

$\frac{5}{x-2}=\frac{9}{3x-6}$ $\qquad$ LCD: $(x-2)(3x-6)$

$\frac{5}{x-2}\frac{(3x-6)}{(3x-6)}=\frac{9}{3x-6}\frac{(x-2)}{(x-2)}$ $\qquad$ Cancel out LCD.

$15x-30=9x-18$

$6x=12$

$x=2$

$NPV: x\neq 2\to extraneous$ roots $\to x=2$ is rejected

No solutions!

7.7 Applications of R.E.

Box Questions:

A company makes boxes (V=1000cm3) with a square base of 15cm. Find height of box. If base length decreases by "x" what is the increase in height?

$V=base\times base\times height$

$1000=15\times 15\times h$

$1000\div 225=h$

$h=4.44cm$

$NOte: \quad h=\frac{Volume}{(base)^2}$

let y be increase in height

$h=4.44+y$

$w=l=(base-x)$

$h=4.44+y=\frac{1000}{(base-x)^2}$

$y=\frac{1000}{(base-x)^2}-4.44$

Speed Questions:

Jack and Jill both travel 300km. Jack's speed is 10times faster than Jill's and took 4.5hours less.
What are their speeds?. $Note: Speed=\frac{Distance}{Time}$

	Dist.	Speed	Time
Jack	300km	10x	300/10x
Jill	300km	x	300/x

$Note: \quad Jack's\ Time=Jill's\ Time-4.5$

$\frac{300}{10x}=\frac{300}{x}-4.5$ $\qquad$ $4.5x=270$

$30=300-4.5x$ $\qquad$ $x=60km/hr$

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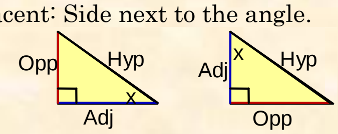
Chapter 8: Trigonometry

SOH-CAH-TOA

Basic $\sin x, \cos x, \& \tan x$ functions can only be used with right triangles.

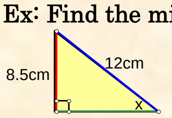
$\sin x = \frac{Opp}{Hyp}$ $\cos x = \frac{Adj}{Hyp}$ $\tan x = \frac{Opp.}{Adj}$

Hypotenuse: The longest side, opposite from the square.
Opposite: Side opposite from the angle.
Adjacent: Side next to the angle.



The opposite and adjacent side will interchange depending on where the angle "x" is.

Ex: Find the missing angle:



$opp = 8.5, hyp = 12$

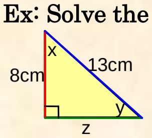
$\sin x = \frac{8.5}{12}$

$x = \sin^{-1}(0.708)$ Inverse!

$x = 45^\circ$

Solving a right triangle means finding all the missing sides and angles

Ex: Solve the right triangle:



Use Pythagorus to find the missing side:

$8^2 + z^2 = 13^2$

$z^2 = 105$

$z = \sqrt{105}$

Use SOH-CAH-TOA to find missing angles

$\cos x = \frac{8}{13}$ $\quad \angle y = 90^\circ - x$

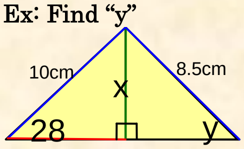
$x = \cos^{-1}\left(\frac{8}{13}\right)$ $\quad \angle y = 90^\circ - 52^\circ$

$x = 52^\circ$ $\quad \angle y = 38^\circ$

8.4: Solving Double Right Triangles

Most questions in this section is about solving problems with two right triangles placed next to each other.

Ex: Find "y"



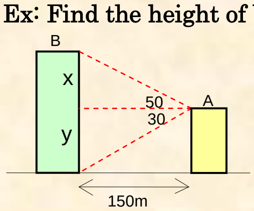
First find "x"

$\sin 28^\circ = \frac{x}{10}$

$4.7cm = x$

Then find "y" $\sin y = \frac{4.7}{8.5} \rightarrow y = \sin^{-1}\left(\frac{4.7}{8.5}\right) \rightarrow y = 33.7^\circ$

Ex: Find the height of building "B"



The height is the sum of $x \& y$: First find "x"

$\tan 50^\circ = \frac{x}{150}$

$178.8m = x$

Then find "y"

$\tan 30 = \frac{y}{150} \rightarrow y = 150(\tan 30) \rightarrow y = 86.6m$

Total Height $= 178.8m + 86.6m = 265.4m$

8.5 Defining $\sin x$ & $\cos x$ of Obtuse Angles

	Q1	Q2	Q3	Q4
Sin x	+	+	−	−
cos x	+	−	+	−
tan x	+	−	−	+

Q2	Q1
S	A
T	C
Q3	Q4

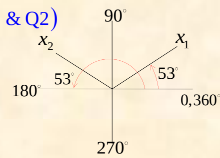
Ex: Find x

$i) \sin x = 0.8$ (positive → Q1 & Q2)

$x = \sin^{-1}(0.8)$

$x_1 = 53^\circ$

$x_2 = 180^\circ - x_1 = 127^\circ$

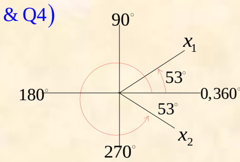


$ii) \cos x = 0.75$ (positive → Q1 & Q4)

$x = \cos^{-1}(0.75)$

$x_1 = 41.4^\circ$

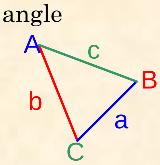
$x_2 = 360^\circ - x_1 = 318.6^\circ$



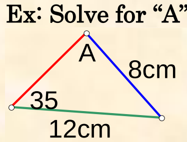
8.7 Sine Law

The Sine Law can be used for solving non-right triangles. Requirements:
One side with an opposite angle

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Ex: Solve for "A"



$\frac{\sin A}{12} = \frac{\sin 35}{8} \rightarrow \sin A = \frac{12 \sin 35}{8}$

$\sin A = 0.86 \rightarrow A = \sin^{-1}(0.86)$

$= 59.4^\circ$

Ex: Solve for "B" given: $a = 5, \angle A = 28^\circ, b = 10$

(Note: "B" is obtuse)

$\frac{\sin 28^\circ}{5} = \frac{\sin b}{10}$ cross mulitply & *simplify*

$2 \sin 28 = \sin b$

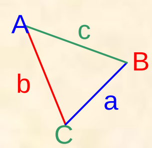
$0.939 = \sin b \rightarrow \sin^{-1}(0.939) = b$

$b = 70^\circ$ (Obtuse!) $\rightarrow b = 180 - 70 = 110^\circ$

8.8 Cosine Law

The cosine law is used when given 2 sides with angle in between and asked to find opposite side of given angle or when given all 3 sides and asked to find any angle.

$$a^2 = b^2 + c^2 - (2bc)\cos A$$
$$b^2 = a^2 + c^2 - (2ac)\cos B$$
$$c^2 = a^2 + b^2 - (2ab)\cos C$$



Ex: *Find "c", given* $\angle C = 55^\circ, a = 9, b = 7$:

$c^2 = 9^2 + 7^2 - 2(63)\cos 55^\circ$

$c^2 = 130 - 126(0.5735)$

$c^2 = 130 - 72.27 = 57.73$

$c = \sqrt{57.73} = 7.6$ *units*

Ex: *Find* $\angle B$, given $a = 10, b = 5, c = 6$:

$5^2 = 10^2 + 6^2 - 2(10)(6)\cos B$

$25 = 136 - 120 \cos B$ Simplify, Isolate "B"

$\frac{-111}{-120} = \cos B$ Inverse!

$22.3^\circ = B$

Chapter 9: Probability

9.2 Sampling Method:

Simple Random Completely random, ie: names in a hat	**Sytematic** ie: every 5th person in a population is chosen
Stratified Random one member selected from different groups	**Cluster** one group is chosen among several groups
Convenience only convenient members are chosen	**Self-Selected** interested members will participate

Box Plots and Sampling

Box plots are for events with binomial outcomes (2 possibilities: ie: T/F, Success or failure).
Desired percentages on the left & number of successes on the bottom. Each box corresponds with a desired percentage.

Sample Size: The number of trials performed.

Ex: If I rolled a dice 20 times, how many times will the number 6 appear?

Desired Percentage $= \frac{1}{6} \simeq 16.6\%$ (Y axis)

Sample Size: n =20

Box Range: (x-axis) 1 to 6.

There is a 90% chance that the number 3 will appear from 1 to 6 times.

Probability:

$$P(x) = \frac{\text{Number of desired outcomes}}{\text{total number of outcomes}}$$

The probability of an event can range of zero to 100%.

Ex: Find the probability of each event:
i) Drawing a **club** from a **deck of cards**

$P(clubs) = \frac{13clubs}{52cards} = \frac{1}{4} = 25\%$

ii)Rolling a 4 or 5 from a die.

$P(4or5) = \frac{2\text{desired outcomes}}{6\text{total outcomes}} = \frac{1}{3} \approx 33\%$

iii) Flipping 2 coins and getting 1 head

$P(1head) = \frac{HT \;\; TH}{HH\;HT\;TH\;TT} = \frac{2}{4} = 50\%$

Expected Value

The amount you expect to win or lose each game on average when playing many times.
Step: Multiply the payoff of each event by its probability. Then add all the results.

Ex: Find the expected value for the following dice game:

Dice	1,2,3	4,5	6
Prize	+0.05	−0.10	+0.25

$E = \left(\frac{3}{6}\right)(0.05) + \left(\frac{2}{6}\right)(-0.10) + \left(\frac{1}{6}\right)(0.25)$

$E = 0.025 + (-0.033) + (0.042)$

$E = \$0.03 \rightarrow$ Expect to win \$0.03 each game.

If you play the game 100times, on average you will win \$3 altogether.