SOLUTION for Pre-Calculus 11 HW 4.5 Discriminant Nature of the Roots, \( D = b^2 - 4ac \)

1. Determine the nature of the roots [ie: Determine how many x-intercepts each quadratic equation has]

<table>
<thead>
<tr>
<th>Equation</th>
<th>Nature of Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 + 5x + 6 = 0 )</td>
<td>There are two distinct real roots</td>
</tr>
<tr>
<td>( 12x^2 + 7x - 3 = 0 )</td>
<td>There are two distinct real roots</td>
</tr>
<tr>
<td>(-2x^2 - 7x + 5 = 0 )</td>
<td>There are two distinct real roots</td>
</tr>
<tr>
<td>( 4x^2 = 13x - 8 )</td>
<td>There are two distinct roots</td>
</tr>
<tr>
<td>( x(7 - 8x) = 10 )</td>
<td>No real roots</td>
</tr>
<tr>
<td>( x(x + 2) = 6 - (x - 3)(2x + 1) )</td>
<td>There are two distinct roots</td>
</tr>
</tbody>
</table>

2. Solve each of the following inequalities:

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 &lt; 16 )</td>
<td>(-4 &lt; x &lt; 4 )</td>
</tr>
<tr>
<td>( x^2 - 25 &gt; 0 )</td>
<td>( x &lt; -5 ) or ( 5 &lt; x )</td>
</tr>
<tr>
<td>( x(3 - x) &lt; 0 )</td>
<td>( x = 0, x = 3 )</td>
</tr>
<tr>
<td>( x(3 - x) &lt; 0 )</td>
<td>( x &lt; 0 ) or ( 3 &lt; x )</td>
</tr>
</tbody>
</table>

3. Determine the value of “k” so that the equation has two equal roots:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 + kx + 25 = 0 )</td>
<td>( k = \pm 10 )</td>
</tr>
<tr>
<td>( kx^2 + 4x + 1 = 0 )</td>
<td>( k = 2, 8 )</td>
</tr>
<tr>
<td>( 0.5x^2 + 3kx + (3k - 4) = 0 )</td>
<td>( k = \frac{4}{3} ) or ( k = \frac{-2}{3} )</td>
</tr>
</tbody>
</table>
4. Determine the value of “k” so that the equation has two different roots:

   a) \( x^2 - kx + 12 = 0 \)
   - To have two distinct roots, the discriminant must be greater than 0.
     \( b^2 - 4ac > 0 \)
     \( k^2 - 4(12) > 0 \)
     \( k^2 - 48 > 0 \)
     \( k < -4\sqrt{3} \text{ or } 4\sqrt{3} < k \)
   - Draw a number line and use test points:

   b) \( 2kx - kx - 1 = 0 \)
   - To have two distinct roots, the discriminant must be greater than 0.
     \( b^2 - 4ac > 0 \)
     \( k^2 - 4(1) > 0 \)
     \( k^2 - 4 < 0 \text{ or } 4 < k \)
   - So as long as \( k \) is between 0 and 4, the quadratic equation will have two distinct roots

5. Determine the value of “k” so that the equation has no real roots:

   a) \( x^2 - kx - 24 = 0 \)
   - To have no real roots, the discriminant must be less than 0
     \( b^2 - 4ac < 0 \)
     \( k^2 - 4(-24) < 0 \)
     \( k^2 + 96 < 0 \)
   - The left side is always positive, because \( k^2 \) is always positive.
   - SO, no matter what “k”, the equation will always have 2 distinct roots

   b) \( 2kx - kx + 8 = 0 \)
   - To have no real roots, the discriminant must be less than 0
     \( b^2 - 4ac < 0 \)
     \( k^2 - 4(8) < 0 \)
     \( k^2 - 32k < 0 \)
     \( k(k - 32) < 0 \)
   - \( 0 < k < 32 \)
   - if “K' is between 0 and 32, the equation will not have any roots

   c) \( x^2 - 3kx - (3k - 8) = 0 \)
   - To have no real roots, the discriminant must be less than 0
     \( b^2 - 4ac < 0 \)
     \( (-3k)^2 - 4(-3k + 8) < 0 \)
     \( 9k^2 + 12k - 32 < 0 \)
     \( (3k + 8)(3k - 4) < 0 \)
   - \( -8 < k < 4 \)
   - \( -\frac{8}{3} < k < \frac{4}{3} \)

6. In order for a quadratic function to be factorable, what value must the discriminant be equal to? Explain:

   This is the quadratic formula: \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \). In order for a quadratic equation to be factorable, both roots must be either an integer or a fraction. Can’t have a radical. So that means the discriminant \( b^2 - 4ac \) needs to be either 0 or a perfect square.

   If the quadratic equation \((x-2)^2 + k = 0\) has two distinct real roots, then what is the range of “k”? (Multiple choice, circle one) Justify your answer.

   a) \( k > 2 \)
   b) \( k < 0 \)
   c) \( k \leq 0 \)
   d) \( k \leq 4 \)

   \( x^2 - 4x + 4 + k = 0 \)
   \( 16 - 4(1)(4 + k) > 0 \)
   \( 16 - 4(4 + k) \)
   \( 16 - 16 - 4k > 0 \)
   \( -4k > 0 \)
   \( k < 0 \)