Name:______________________________________ Date:____________________________

Pre-Calculus 11: HW 1.4 Geometric Series $S_n = \dfrac{a(1-r^n)}{1-r}$

1. Given each geometric sequence, indicate the values of the first term "a", number of terms "n", and common ratio "r"

a) $36+18+9+4.5+2.25+1.125$

$a = 36$ $r = 0.5$ $n = 6$

b) $1+2+4+8+16+32+64+128$

$a = 1$ $r = 2$ $n = 7$

c) $-5+10+(-20)+40+(-80)+......+t_{11}$

$a = -5$ $r = -2$ $n = 11$

d) $0.125+0.25+0.5+1+2+4+8+16+32$

$a = 0.125$ $r = 2$ $n = 9$

e) $4+(-4)+4+(-4)+4+(-4)+4+(-4)$

$a = 4$ $r = -1$ $n = 8$

f) $\frac{5}{4}+\frac{5}{2}+5+...+40$

$40 = \frac{5}{4}(2)^{n-1}$

$32 = 2^{n-1}$ $a = -5/4$ $r = 2$ $n = 6$

$5 = n-1$

$6 = n$

g) $\frac{2}{3}+2+6+18+....+486$

$486 = \frac{2}{3}(3)^{n-1}$

$729 = 3^{n-1}$ $a = 2/3$ $r = 3$ $n = 7$

$6 = n-1$

$7 = n$

h) $\frac{27}{16}+\frac{9}{4}+3+......+\frac{64}{9}$

$\frac{64}{9} = \frac{27}{16}\left(\frac{4}{3}\right)^{n-1}$

$\frac{1024}{243} = \left(\frac{4}{3}\right)^{n-1}$ $a = 27/16$ $r = 4/3$ $n = 6$

$5 = n-1$

$6 = n$

2. Given each of the following series, find the sum

a) $2.5+5+10+20+40+....+t_8$

$a = 2.5, r = 2, n = 8$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{2.5(1-2^8)}{1-2}$

$= \dfrac{2.5(1-256)}{(-1)}$

$= 637.5$

b) $8+12+18+27+40.5+.....t_9$

$a = 8, r = \frac{3}{2}, n = 9$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{8(1-1.5^9)}{1-1.5}$

$= \dfrac{8(1-38.443359375)}{(-0.5)}$

$= 599.09375$

c) $0.25 + 0.50 + 1.0 + 2.0 + 4.0 + \ldots + t_{10}$

$a = 0.25, r = 2, n = 10$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{0.25(1-2^{10})}{1-2} = \frac{0.25(-1023)}{(-1)} = 255.75$$

d) $\frac{2}{3} + \frac{-1}{3} + \frac{1}{6} + \frac{-1}{12} + \frac{1}{24} + \ldots\ t_7$

$a = 2/3, r = -0.5, n = 7$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{\frac{2}{3}\left(1-(-0.5)^7\right)}{1-(-0.5)} = \frac{\frac{2}{3}(1-0.0078125)}{1.5} = 0.44097222222$$

e) $4 + 8 + 16 + 32 + 64 \ldots + 2048$

$a = 4, r = 2, t_n = 2048, n = ?$

$t_n = a \times r^{n-1}$

$2048 = 4 \times (2)^{n-1}$

$512 = 2^{n-1}$

$9 = n - 1$

$10 = n$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{4(1-2^{10})}{1-2} = \frac{4(1-1024)}{(-1)} = 4092$$

f) $3 + 9 + 27 + 81 + 243 + \ldots + 19683$

$a = 3, r = 3, t_n = 19683, n = ?$

$t_n = a \times r^{n-1}$

$19683 = 3 \times (3)^{n-1}$

$6561 = 3^{n-1}$

$8 = n - 1$

$9 = n$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{3(1-3^9)}{1-3} = \frac{3(1-19683)}{(-2)} = 29523$$

g) $24 + 12 + 6 + 3 + \ldots + \frac{3}{16}$

$a = 24, r = 0.5, t_n = 3/16, n = ?$

$t_n = a \times r^{n-1}$

$\frac{3}{16} = 24 \times (0.5)^{n-1}$

$\frac{1}{128} = (0.5)^{n-1}$

$7 = n - 1$

$8 = n$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{24(1-0.5^8)}{1-2} = \frac{24(1-1024)}{(-1)} = 23.90625$$

h) $\frac{64}{27} + \frac{32}{9} + \frac{16}{3} + 8 + \ldots + 40.5$

$a = \frac{64}{27}, r = \frac{3}{2}, t_n = 40.5, n = ?$

$t_n = a \times r^{n-1}$

$\frac{81}{2} = \frac{64}{27} \times (1.5)^{n-1}$

$\frac{2187}{128} = \left(\frac{3}{2}\right)^{n-1}$

$7 = n - 1$

$8 = n$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{\frac{64}{27}\left(1-\frac{3}{2}^7\right)}{1-1.5} = \frac{\frac{64}{27}(1-17.0859375)}{(-0.5)} = 76.2592592593$$

i) $a = 3.5, r = 0.5, n = 10, S_{10} = ?$

$a = 3.5, r = 0.5, n = 10$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{3.5(1-0.5^{10})}{1-0.5} = \frac{3.5(0.9990234375)}{0.5} = 6.9931640625$$

j) $a = -4, r = -2, n = 6, S_6 = ?$

$a = -4, r = -2, n = 6$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{-4\left(1-(-2)^6\right)}{1+2} = \frac{-4(-63)}{(3)} = 84$$

k) $a=\frac{27}{32},\ r=\frac{2}{3},\ n=8, S_8=?$ $a=\frac{27}{32}, r=\frac{2}{3}, n=8$ $S_n=\frac{a(1-r^n)}{1-r}$ $=\frac{\frac{27}{32}\left(1-\frac{2}{3}^8\right)}{1-\frac{2}{3}}$ $=\frac{\frac{27}{32}(0.96098155768)}{\frac{1}{3}}$ $=2.4324845679$	L) $a=125, r=0.2, n=7, S_7=?$ $a=125, r=0.2, n=7$ $S_n=\frac{a(1-r^n)}{1-r}$ $=\frac{125(1-0.2^7)}{1-0.2}$ $=\frac{125(0.9999872)}{0.8}$ $=156.248$

3. Given each geometric series, find the value of the missing term:

a) $S_6=341.25, r=\frac{1}{4}, a=?$ $a=?, r=0.25, n=6, S_6=341.25$ $S_n=\frac{a(1-r^n)}{1-r}$ $341.25=\frac{a(1-0.25^6)}{1-0.25}$ $341.25=\frac{a(0.99975585937)}{0.75}$ $256=a$	b) $S_6=567, r=\frac{1}{2}, a=?$ $a=?, r=0.5, n=6, S_6=567$ $S_n=\frac{a(1-r^n)}{1-r}$ $567=\frac{a(1-0.5^6)}{1-0.5}$ $567=\frac{a(0.984375)}{0.5}$ $288=a$
c) The sum of the first 8 terms of a geometric series is 1020 with $r=-2$. Determine the first term. $a=?, r=-2, n=8, S_8=1020$ $S_n=\frac{a(1-r^n)}{1-r}$ $1020=\frac{a(1-(-2)^8)}{1-(-2)}$ $1020=\frac{a(-255)}{3}$ $12=a$	d) $S_5=100, S_4=87, t_5=?$ $S_5=100, S_4=87, t_5=?$ $S_5=t_1+t_2+t_3+t_4+t_5$ $S_4=t_1+t_2+t_3+t_4$ $S_5-S_4=t_4$ $100-87=t_4$ $13=t_4$

4. The sum of the 1st and 2nd term of a geometric sequence is 4 and the sum of the 3rd and 4th term is 36. Determine the sum of the first 8 terms.

$a + ar = 4 \qquad ar^2 + ar^3 = 36$

Factor out any common factors from both equations to simplify it:

$a(1+r) = 4 \qquad ar^2(1+r) = 36$

Divide the two equations to solve for the common ratio "r"

$$\frac{ar^2(1+r)}{a(1+r)} = \frac{36}{4}$$
$$\frac{ar^2}{a} = 9$$
$$r^2 = 9$$
$$r = \pm 3$$

There will now be two different possible answers since the common ratio has two different values (+) and (-). Now solve for "a" for each common ratio:

$$a(1+r) = 4 \qquad a(1+r) = 4$$
$$a(1+(-3)) = 4 \qquad a(1+(3)) = 4$$
$$a(-2) = 4 \qquad a(4) = 4$$
$$a = -2 \qquad a = 1$$

Since we have two different "a" values, that means there will be two different sequences and two different sums:

First Sum (a=-2)

$$S_8 = \frac{a(1-r^n)}{1-r}$$
$$S_8 = \frac{-2\left(1-(-3)^8\right)}{1-(-3)}$$
$$S_8 = \frac{-2(1-6561)}{4}$$
$$S_8 = 3280$$

For the second sum (a=1)

$$S_8 = \frac{a(1-r^n)}{1-r}$$
$$S_8 = \frac{1\left(1-(3)^8\right)}{1-(3)}$$
$$S_8 = \frac{1(1-6561)}{-2}$$
$$S_8 = 3280$$

5. Challenge: In a geometric series, $S_7 = 381$ and $S_6 = 189$, what is the value of the common ratio? Please show all your work:

There are two separate solutions. In my opinion, the first one is better.

Solution 1:

Since $S_6 = 189$ $\quad 189 = \dfrac{a(r^6 - 1)}{r-1}$ and $S_7 = 381$ $\quad 381 = \dfrac{a(r^7-1)}{r-1}$, from here there are usually a few options. You can either, "add", "subtract", or "divide" the two equations to eliminate some variables. So, I decided to divide them:

$$\frac{S_7}{S_6} = \frac{381}{189} \rightarrow \frac{381}{189} = \frac{a(r^7-1)}{r-1} \times \frac{r-1}{a(r^6-1)}$$ Cancel and simplify anything possible:

$$\rightarrow \frac{127}{63} = \frac{(r^7-1)}{(r^6-1)}$$ after we simplify, we know that: $2^7 - 1 = 127$ and $2^6 - 1 = 63$

Therefore, we conclude that $r = 2$

2nd Method: (not as good)

One way to start this question off is to write out the series:

$$S_7 = a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6 = 381$$
$$S_6 = a + ar + ar^2 + ar^3 + ar^4 + ar^5 = 192$$

Both series can factor out the constant "a". Both sums have a greatest common factor of "3"

$$a(1 + r + r^2 + r^3 + r^4 + r^5 + r^6) = 3(127)$$
$$a(1 + r + r^2 + r^3 + r^4 + r^5) = 3(63)$$

Then lets assume that a=3

If we subtract both series then:

$$\begin{array}{r} a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6 \\ \underline{-(a + ar + ar^2 + ar^3 + ar^4 + ar^5)} \end{array} = \begin{array}{r} 381 \\ \underline{-(189)} \end{array}$$ then substitute a=3

$$ar^6 = 192$$
$$3r^6 = 3(64)$$
$$r^6 = 64$$
$$r = \pm 2$$

The problem with this solution is that we assume "a" is equal to 3. With 2 variables and only one equation, it is possible to have more than one solution.