Name:_____________________________ Date:___________________________

Pre Calculus 11 HW: Section 3.3 Completing the Square

1. Indicate what value should be added to the trinomial so that the equation could be a perfect trinomial:

a) $x^2+(?)+9$ $x^2+6x+9=(x+3)(x+3)$ $(?)=6x$	b) $x^2+8x+(?)$ $x^2+8x+16=(x+4)(x+4)$ $(?)=16$
c) $(?)-2x+1$ $x^2-2x+1=(x-1)(x-1)$ $(?)=x^2$	d) $x^2-(?)+81$ $x^2-18x+81=(x-9)(x-9)$ $(?)=18x$
e) $x^2-15x+(?)$ $x^2-15x+\frac{225}{4}=\left(x-\frac{15}{2}\right)\left(x-\frac{15}{2}\right)$ $(?)=\frac{225}{4}$	f) $x^2+17x+(?)$ $x^2+17x+\frac{289}{4}=\left(x+\frac{17}{2}\right)\left(x+\frac{17}{2}\right)$ $(?)=\frac{289}{4}$
g) $4x^2+4x+(?)$ $4x^2+4x+1=(2x+1)(2x+1)$ $(?)=1$	h) $9x^2-(?)+1$ $9x^2-6x+1=(3x-1)(3x-1)$ $(?)=6x$

2. Given each equation in to general form: $y=a(x-p)^2+q$ by completing the square. Show all your steps:

a) $y=x^2+4x-20$ $y=(x^2+4x)-20$ $y=(x^2+4x+4-4)-20$ $y=(x^2+4x+4)-24$ $y=(x+2)(x+2)-24$ $y=(x+2)^2-24$	b) $y=x^2-8x-20$ $y=(x^2-8x)-20$ $y=(x^2-8x+16-16)-20$ $y=(x^2-8x+16)-36$ $y=(x-4)(x-4)-36$ $y=(x-4)^2-36$
c) $y=-x^2-14x-15$ $y=(-x^2-14x)-15$ $y=-(x^2+14x)-15$ $y=-(x^2+14x+49-49)-15$ $y=-(x+7)(x+7)+49-15$ $y=-(x+7)^2+34$	d) $y=4x^2+20x-12$ $y=(4x^2+20x)-12$ $y=4(x^2+5x)-12$ $y=4(x^2+5x+6.25-6.25)-12$ $y=4(x^2+5x+6.25)-25-12$ $y=4(x+2.5)^2-37$

e) $y=2x(x-5)$ $y=2x(x-5)$ $y=2(x^2-5x)$ $y=2(x^2-5x+6.25-6.25)$ $y=2(x^2-5x+6.25)-12.5$ $y=2(x-2.5)^2-12.5$	f) $y=3x^2+6x+10$ $y=(3x^2+6x)+10$ $y=3(x^2+2x)+10$ $y=3(x^2+2x+1-1)+10$ $y=3(x^2+2x+1)-3+10$ $y=3(x+1)^2+7$
g) $y=-2x^2-15x+100$ $y=(-2x^2-15x)+100$ $y=-2\left(x^2+\frac{15}{2}x\right)+100$ $y=-2\left(x^2+\frac{15}{2}x+\frac{225}{16}-\frac{225}{16}\right)+100$ $y=-2\left(x^2+7.5x+\frac{225}{16}\right)+\frac{225}{8}+100$ $y=-2\left(x+\frac{15}{2}\right)^2+\frac{225}{8}+\frac{800}{8}$ $y=-2\left(x+\frac{15}{2}\right)^2+\frac{1025}{8}$	h) $y=-3x^2+18x+50$ $y=(-3x^2+18x)+50$ $y=-3(x^2-6x)+50$ $y=-3(x^2-6x+9-9)+50$ $y=-3(x^2-6x+9)+27+50$ $y=-3(x-3)^2+77$
e) $y=-7x^2+182x+100$ $y=(-7x^2+182x)+100$ $y=-7(x^2-26x)+100$ $y=-7(x^2-26x+169-169)+100$ $y=-7(x^2-26x+169)+7(169)+100$ $y=-7(x-13)^2+1283$	f) $y=\frac{1}{2}x^2+8x-30$ $y=\left(\frac{1}{2}x^2+8x\right)-30$ $y=\frac{1}{2}(x^2+16x)-30$ $y=\frac{1}{2}(x^2+16x+64-64)-30$ $y=\frac{1}{2}(x^2+16x+64)-32-30$ $y=\frac{1}{2}(x+8)^2-62$

3. Two numbers have a difference of 10. Their product is a minimum. Determine the numbers

   Let the numbers be “x” and “y”

   $y-x=10$

   $y=10+x$

   The “product” means you multiply the two numbers together:

$$P = x(y)$$
$$P = x(10 + x)$$
$$P = x^2 + 10x$$
$$P = (x^2 + 10x + 25) - 25$$
$$P = (x+5)^2 - 25$$

Looking at the equation, the vertex is at (-5,-25). This means that the value of "x" is -5 and the minimum product is -25

To find the other value "y", use the original equation : $y = 10 + x$

$$y = 10 + (-5)$$
$$y = 5$$

4. The sum of two natural numbers is 12. Their product is a maximum. Determine the numbers

Sum mean you add the two numbers together: $x + y = 12$

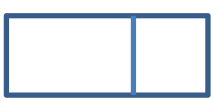
$$y = 12 - x$$

$$P = x(y)$$
$$P = x(12 - x)$$
$$P = -x^2 + 12x$$
$$P = -(x^2 - 12x)$$
$$P = -(x^2 - 12x + 36 - 36)$$
$$P = -(x-6)^2 + 36$$

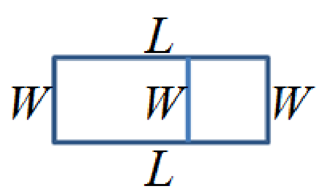
The vertex is at $(6, 36)$. The value of "x" is 6 and the maximum product is 36. To find *the* other value "y", use the original equation:

$$y = 12 - x$$
$$y = 12 - 6 = 6$$

5. A rectangular area is enclosed by a fence and separated into 2 rectangular regions as shown. With 800m of fencing, what is the maximum area that could be enclosed. Find the dimensions of the enclosed area.



Write down the dimensions on the diagram:



Since we have a perimeter of 800meters, we get the first equation: $3W + 2L = 800$

$$L = -1.5W + 400$$

The area is the product of the length and width.

$A = L \times W$

$A = (-1.5W + 400)W$

$A = (-1.5W^2 + 400W)$

$A = -1.5\left(W^2 - \frac{800}{3}w\right)$

$A = -1.5\left(W^2 - \frac{800}{3}w + \frac{160000}{9}\right) + 1.5\left(\frac{160000}{9}\right)$

$A = -1.5\left(W - \frac{400}{3}\right)^2 + \frac{80000}{3}$

the vertex is $\left(\frac{400}{3}, \frac{80000}{3}\right)$

This means the width is $\frac{400}{3}$ and

the maximum area is $\frac{80000}{3}$

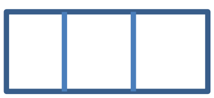
To find the length, use the original formula or just take the area and divide by the width:

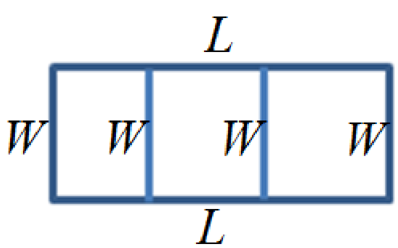
$A = W \times L$

$\frac{80000}{3} = \frac{400}{3} \times L$

$200 = L$

6. Suppose the rectangular fence is to be separated into 3 rectangular regions as shown. Again, with 800m of fencing, find the maximum area that could be enclosed. Find the dimensions of the enclosed area.





Using the diagram, we can get the first formula: $2L + 4W = 800$. Isolate one of the variables, take your pick: $L = -2W + 400$

Then use the area formula and find your vertex:

$A = L \times W$

$A = (-2W + 400)W$

$A = -2(W^2 - 200W)$

$A = -2(W^2 - 200W + 10000 - 10000)$

$A = -2(W - 100)^2 + 20000$

The width is 100meters, and the maximum area is 20,000m^2

To find the length, divide the area by the width:

$\frac{Area}{Width} = Length$

$\frac{20{,}000m^2}{100m} = L$

$200m = L$

7. A company that charters a boat for tours around Vancouver Island can sell 200 tickets at $50 each. For every $10 increase in the ticket price, 5 fewer tickets will be sold. What selling price will provide the maximum revenue? What is the maximum revenue?

8. A Broadway musical sells 400 tickets each day at $30 per ticket. For every increase of $3.00, they lose 20 sales. What should their ticket price be to yield the maximum revenue?

9. A company sells its bikes at $300 each and can sell 70 in a season. For every $25 increase in the price, the number sold drops by 10. What price will yield the maximum revenue?

10. A farmer wants to make a rectangular corral by using his barn wall as one of the sides of the corral. If the farmer has only 60m of fence, what length for the rectangular corral would maximize the area?

11. Challenge: This one is super hard. The parabola $y = f(x) = x^2 + bx + c$ has vertex "P" and the parabola $y = g(x) = -x^2 + dx + e$ has vertex "Q", where "P" and "Q" are distinct points. The two parabolas also intersect at "P" and "Q". Prove that $2(e-c) = bd$.