Pre Calculus 11 HW: Section 3.2 Quadratic Functions  \( y = ax^2 + bx + c \)

1. For each of the following quadratic functions find the coefficients “\(a,b,c\)” and then find i) the Coordinates of the Vertex and the iii) Domain and Range

<table>
<thead>
<tr>
<th>Quadratic Function</th>
<th>Coefficients (a,b,c)</th>
<th>Vertex ((h,k))</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (y = x^2 + 3x - 18)</td>
<td>(a = 1), (b = 3), (c = -18)</td>
<td>(P = -\frac{b}{2a} = -\frac{3}{2(1)})( Q = c - \frac{b^2}{4a} = -18 - \frac{9}{4} )</td>
<td>(x \in \mathbb{R})</td>
<td>(y \geq -\frac{81}{4})</td>
</tr>
<tr>
<td>b) (y = 2x^2 - x - 2)</td>
<td>(a = 2), (b = -1), (c = -2)</td>
<td>(P = -\frac{b}{2a} = -\frac{1}{2(2)})( Q = c - \frac{b^2}{4a} = -2 - \frac{1}{8} )</td>
<td>(x \in \mathbb{R})</td>
<td>(y \geq -\frac{17}{8})</td>
</tr>
<tr>
<td>c) (y = -x^2 - 12x - 35)</td>
<td>(a = -1), (b = -12), (c = -35)</td>
<td>(P = -\frac{b}{2a} = -\frac{-12}{2(-1)})( Q = c - \frac{b^2}{4a} = -35 - \frac{144}{-4} )</td>
<td>(x \in \mathbb{R})</td>
<td>(y \leq 1)</td>
</tr>
<tr>
<td>d) (y = 6x^2 + 13x - 5)</td>
<td>(a = 6), (b = 13), (c = -5)</td>
<td>(P = -\frac{b}{2a} = -\frac{13}{2(6)})( Q = c - \frac{b^2}{4a} = -5 - \frac{169}{4(6)} )</td>
<td>(x \in \mathbb{R})</td>
<td>(y \geq -\frac{289}{24})</td>
</tr>
<tr>
<td>e) (y = 2x(x - 4))</td>
<td>(a = 2), (b = -8), (c = 0)</td>
<td>(P = -\frac{b}{2a} = -\frac{-8}{2(2)})( Q = c - \frac{b^2}{4a} = 0 - \frac{64}{4(2)} )</td>
<td>(x \in \mathbb{R})</td>
<td>(y \geq -8)</td>
</tr>
<tr>
<td>f) (y = (2x - 1)(3x + 4))</td>
<td>(a = 6), (b = 5), (c = -4)</td>
<td>(P = -\frac{b}{2a} = -\frac{5}{2(6)})( Q = c - \frac{b^2}{4a} = -4 - \frac{25}{4(6)} )</td>
<td>(x \in \mathbb{R})</td>
<td>(y \geq -\frac{121}{24})</td>
</tr>
</tbody>
</table>
2. Find the coordinates of the vertex and then graph it on the grid provided. Label the vertex, axis of symmetry, y-intercept on the graph.

a) \( f(x) = x^2 + 7x + 10 \)

b) \( y = x^2 - 6x + 8 \) \( x = 3 \)

c) \( y = x^2 + 4x + 8 \)

d) \( y = 2x^2 + 16x + 26 \)

e) \( y = -x^2 - 4x - 3 \)

f) \( y = -3x^2 - 18x - 25 \)
3. A pebble is thrown from a bridge into a river at height “h” meters above the river. Let “t” be the number of seconds after the release. If the height of the pebble is given by the equation: \( h(t) = -4.9t^2 + 10t + 65 \), then:

a) How high is the pebble after 3 seconds?

\[
\begin{align*}
  h(t) &= -4.9t^2 + 10t + 65 \\
  h(3) &= -4.9(9) + 10(3) + 65 \\
  h(3) &= -44.1 + 30 + 65 \\
  h(3) &= 50.9m
\end{align*}
\]

The pebble will be 50.9 meter above ground.

b) What is the vertex of the equation? What does the vertex represent?

\[
\begin{align*}
  p &= \frac{-b}{2a} = \frac{-10}{2(-4.9)} \\
  q &= c - \frac{b^2}{4a} = 65 - \frac{100}{4(-4.9)} \\
  p &= 1.0204s \\
  q &= 70.102m
\end{align*}
\]

The vertex is (1.0204, 70.102). When the time is 1.0204, the pebble will reach the maximum height of 70.102 meters above ground.

c) What is the domain and range of this scenario and what does it represent?

The domain is \( 0 \leq x \leq 4.8082s \) because the pebble will hit the ground after 4.8082s.

The range is \( 0 \leq y \leq 70.102 \) because the height of the pebble is lowest at the ground when \( h=0 \) and highest when the height is at 70.102m.

NOTE: There are several ways to find the x-intercept: CTS, convert the equation into vertex form and then make \( y=0 \) and solve for “x”. Or use the quadratic formula: 

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ or use the Ti-83 to find the zeroes(x-intercept)}
\]

\[
h(t) = -4.9t^2 + 10t + 65
\]

\[
x = \frac{-10 \pm \sqrt{10^2 - 4(-4.9)65}}{2(-4.9)} = \frac{-10 \pm \sqrt{1374}}{-9.8} = \frac{-10 \pm 37.07}{-9.8} = 4.80s \text{ or } -2.76s
\]
d) What is the y-intercept and what does it represent?

The y-intercept is (0, 65m). When time is zero, the height of the pebble is at 65m. This is the height at which the thrower is at when he/she throws the pebble.

4. Tom throws a football from the top of his building. The height of the ball is given by the formula:

\[ h(t) = -3t^2 + 60t + 132 \]

where “h” is the height of the football and “t” is the number of seconds after the throw.

a) Draw a graph for this scenario and then find the vertex of this equation? Show your work algebraically.

\[ a = -3 \quad b = 60 \quad c = 132 \]

\[ P = -\frac{b}{2a} = -\frac{60}{2(-3)} = 10 \]

\[ Q = 132 - \frac{3600}{4(-3)} = 132 + 300 \]

\[ P = 10 \quad Q = 432 \]

Vertex \((10, 432)\)

b) What is the domain and range of this scenario? Explain it in the context of this question:

The domain is \(0 \leq t \leq 22\) s because the football will hit the ground after 22s.

\[ h(t) = -3t^2 + 60t + 132 \]

\[ 0 = -3(t^2 - 20t - 44) \]

\[ 0 = -3(t - 22)(t + 2) \]

\[ 0 = t - 22 \quad 0 = t + 2 \]

\[ t = 22s \quad t = -2s \]

The range is \(0 \leq h(t) \leq 432\) because the maximum height is at 432m.

c) When will the ball be falling to 150m?

The pebble will be falling at 50m when the time is 21.284s.
5. Suppose you have 100 m of fencing that will be used to build a rectangular fence around your house
   a) Write a quadratic function in standard form to represent the area of the rectangular lot

   \[ 2L + 2W = 100 \]
   \[ L + W = 50 \]
   \[ L = 50 - W \]

   \[ A = L \times W \]
   \[ A = (50 - W)W \]
   \[ A = -W^2 + 50W \]

   b) What are the coordinates of the vertex? What does the coordinate represent?

   \[ a = -1 \quad b = 50 \quad c = 0 \]

   \[ P = -\frac{50}{2(-1)} = 25 \]
   \[ Q = c - \frac{b^2}{4a} = 0 - \frac{2500}{4(-1)} = 625 \]

   The vertex represents the maximum area of the rectangular box. It occurs when the width is 25m, at the y-coordinate is the maximum area of 625m². To find the Length: \( L = 50 - w = 25m \)

   c) Sketch the graph of the function

   [Graph]

   d) Determine the domain and range:

   The domain is \( 0 \leq w \leq 50m \). The smallest width is zero, and the maximum is when the width is 50 leaving now fencing for the length.

   The range is \( 0 \leq A \leq 625m^2 \) because the maximum area is 625 and the smallest area is 0.
6. If the quadratic equation \((x - 2)^2 + k = 0\) has two distinct real roots, then what is the range of “k”?

(Multiple choice, circle one) Justify your answer.

a) \(k > 2\)  

b) \(k < 0\)  

c) \(k \leq 0\)  

d) \(k \leq 4\)

To have two distinct roots means you need to have 2 x-intercepts. In order for a graph that is opening up to have 2 x-intercepts, it needs to be shifted down. Therefore, to move then, then “k” needs to be negative. Therefore \(k < 0\) (B)

7. Point “A” is the vertex of the parabola \(y = x^2 + 2\), point “B” is the vertex of the parabola \(y = x^2 - 6x + 7\), and “O” is the origin. Determine the area of \(\triangle AOB\).

This one is easy. Just find the base and height of the triangle to get the area. Have fun!!