HW SOL 2.3b

November 8, 2016 1:29 PM

Name:_____ Date:_____

Pre-Calculus 11: HW 2.3b Ambiguous Case of Sine Law

1. Given each equation, solve for all values of θ where $0 \le \theta \le 360^\circ$. Note: There are two angles!!

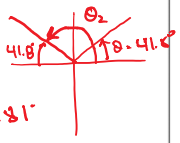
a) $\sin\theta = \frac{2}{3}$ Q1, Q2

$\sin^{-1}\left(\frac{2}{3}\right) = \theta$

$41.81^\circ = \theta_1$

$\theta_2 = 180 - 41.81$

$= 138.2^\circ$



$\theta_1 = 41.8^\circ$ $\theta_2 = 138.2^\circ$

b) $\sin\theta = \frac{4}{5}$ Q1, Q2

$\theta = \sin^{-1}\left(\frac{4}{5}\right)$

$\theta_1 = 53.13^\circ$

$\theta_2 = 180 - 53.13^\circ$

$= 126.87^\circ$

$\theta_1 = 53.13^\circ$ $\theta_2 = 126.87^\circ$

c) $\sin\theta = -0.55$ Q3, Q4

$\theta = \sin^{-1}(-0.55)$

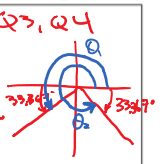
$\theta_1 = -33.367^\circ$

$\theta_1 = 360 - 33367^\circ$

$= 326.63^\circ$

$\theta_2 = 180 + 33.367$

$= 213.37^\circ$



$\theta_1 = 213.37$ $\theta_2 = 326.63^\circ$

d) $\sin\theta = \frac{-\sqrt{2}}{2}$ Q3, Q4

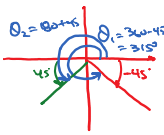
$\theta = \sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

$\theta_1 = -45^\circ$

$\theta_1 = 360 - 45^\circ$

$= 315^\circ$

$\theta_2 = 180 + 45^\circ$



$\theta_1 = 315^\circ$ $\theta_2 = 225^\circ$

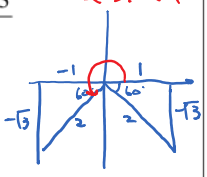
e) $\sin\theta = \frac{-\sqrt{3}}{2}$ Q3, Q4

$\theta_1 = 180 + 60^\circ$

$= 240^\circ$

$\theta_2 = 360 - 60^\circ$

$= 300^\circ$



$\theta_1 = 240^\circ$ $\theta_2 = 300^\circ$

f) $\sin\theta = \frac{4}{\sqrt{7}}$ Q1, Q2.

$\sin\theta = \frac{opp}{hyp}$ hyp ≥ opp b/c hyp must be the longest side.

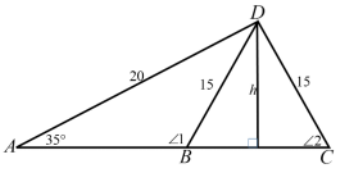
$4 > \sqrt{7}$ ← no contradiction!

No soln!

$\theta_1 =$ ____ $\theta_2 =$ ____

2. Given each triangle, find the missing values and show all your work

a) Find the value of $\angle 1$, $\angle 2$, h, BC, and AB



① $\frac{\sin\angle 1}{20} = \frac{\sin 35^\circ}{15}$

$\sin\angle 1 = \frac{20(\sin 35^\circ)}{15}$

$\angle 1 = \sin^{-1}\left(\frac{4}{3}\cdot\sin 35^\circ\right)$

$\angle 1 = 49.886^\circ$ [$\angle 1$ is OBTUSE]

$\therefore \angle 1 = 180 - 49.886^\circ$

$\angle 1 = 130.11^\circ$

② $\angle 2 = 180 - 130.11^\circ$

$= 49.886^\circ$

③ $\sin 49.886^\circ = \frac{h}{15}$

$11.472 = h$

④ $15^2 - h^2 = BC^2$

$225 - 131.5959 = BC^2$

$93.404 = BC^2$

$9.66 = BC$

⑤ $\angle ADB = 180 - 130.11 - 35^\circ$

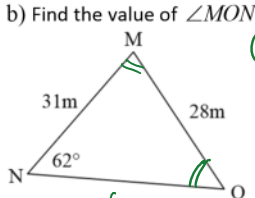
$= 14.89^\circ$

$\frac{\sin 35^\circ}{15} = \frac{\sin 14.89^\circ}{AB}$

$AB = \frac{15(\sin 14.89^\circ)}{\sin 35^\circ}$

$\angle 1 = 130.11^\circ$ $\angle 2 = 49.9^\circ$ $h = 11.472$ $BC = 9.66$ $AB = 6.72$

b) Find the value of $\angle MON, \angle OMN$, and $\overline{ON}$



① $\frac{\sin 62^\circ}{28} = \frac{\sin \angle MON}{31}$

$\frac{31 \cdot \sin 62}{28} = \sin \angle MON$

$\sin^{-1}(\quad) = \angle MON$

$77.83^\circ = \angle MON$

$102.16^\circ = \angle MON$

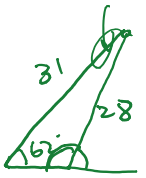
② $\frac{\sin 40.17}{ON} = \frac{\sin 62}{28}$

$\frac{28 \cdot \sin 40.17^\circ}{\sin 62^\circ} = ON$

$\_\_\_\_\_ = ON$

$\frac{28 \times \sin 15.83^\circ}{\sin 62^\circ} = ON$

$\_\_\_\_\_ = ON$



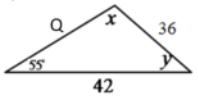
$\angle MON = 77.83^\circ$ (ACUTE) $\quad \angle OMN = 40.17^\circ$ $\quad ON = 20.45$

$180 - 62 - 77.83$

$\angle MON = 102.16^\circ$ (OBTUSE) $\quad \angle OMN = 15.83^\circ$ $\quad ON = 8.6$

$180 - 62 - 102.16$

e)



$\angle x$ obtuse:

$180 - 72.87$

$= 107.13^\circ$

$\angle y$ Acute.

$180 - 72.87 - 55^\circ$

$= 52.13^\circ$

$\angle y$ obtuse:

$180 - 55 - 107.13$

$= 17.87^\circ$

$\angle x$: $\frac{\sin 55}{36} = \frac{\sin x}{42}$

$\sin x = \frac{\sin 55}{36} \times 42$

$\sin x = 0.9556$ $\quad \angle x = 72.87^\circ$

Q Acute:

$\frac{\sin 55}{36} = \frac{\sin 52.13}{Q}$

$Q = \frac{\sin 52.13}{0.02275...}$

$Q = 34.69$

Q obtuse:

$\frac{\sin 55}{36} = \frac{\sin 17.87}{Q}$

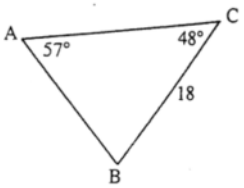
$Q = \frac{\sin 17.87}{0.02275...}$

$Q = 19.48$

$\angle x = 72.87^\circ$ (ACUTE) $\quad \angle y = 52.13^\circ$ $\quad Q = 34.69$

$\angle x = 107.13^\circ$ (OBTUSE) $\quad \angle y = 17.87^\circ$ $\quad Q = 19.48$

3. Find the area of the following triangle. Note the area of a triangle is $A = b \times h \times 0.5$:



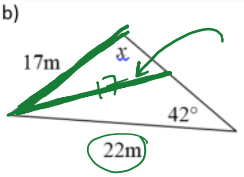
4. Given each of the following triangles, indicate whether if there would be an ambiguous case. State the reason why or why not: Solve for "x".

a)

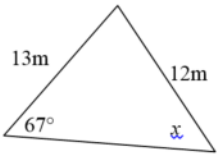
3RD ∠ is Fixed

NOT AMBIGUOUS

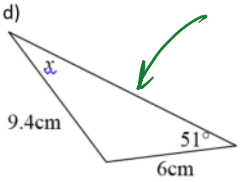
b)



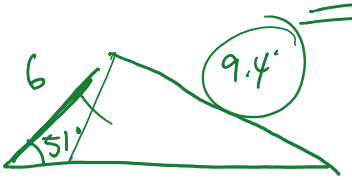
c)



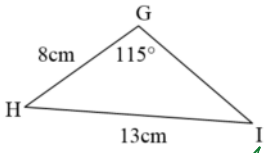
d)



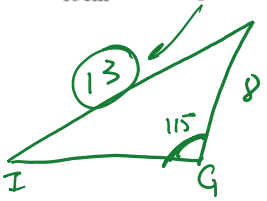
NOT AMBIGUOUS



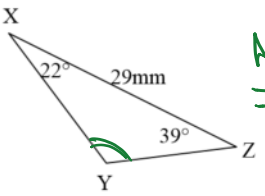
e)



NOT.



f)



AMB

5. A lighthouse at point Q is 20 km from a yacht at point R and 16 km from a sailboat at point S. From the yacht, the lighthouse and the sailboat are separated by an angle of 39°

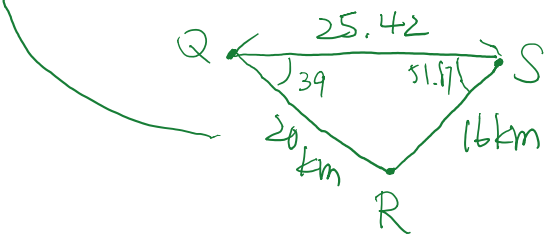
a) Is it necessary to consider the ambiguous case? Explain.

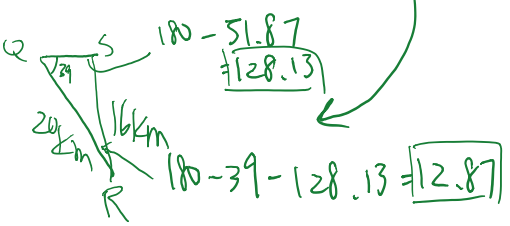
Yes, cause it is ambiguous.

b) Sketch all possible diagrams for this situation.

$\frac{\sin 39}{16} = \frac{\sin(S)}{20}$ $\sin^{-1}(0.7866...) = 51.87$

$\sin(S) = \frac{\sin 39 \times 20}{16} = 0.7866...$ $180 - 39 - 51.87 = 89.13$





$\frac{\sin 89.13}{QS} = \frac{\sin 39}{16}$

$QS = 25.42$ — solution ①

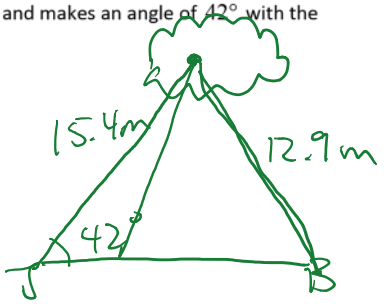
$\frac{\sin 39}{16} = \frac{\sin 12.87}{x}$

$x = \frac{\sin 12.8 \times 16}{\sin 39}$

$= 5.63$ — solution ②

c) Determine all possible the distances from the yacht to the sailboat, to the nearest tenth of a kilometre.

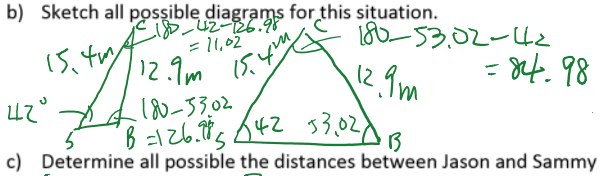
6. Jason and Sammy are part of a scientific team studying clouds. The team is about to launch a weather balloon into an active part of the cloud. Jason's rope is 15.4 m long and makes an angle of 42° with the ground. Belle's rope is 12.9 m long.



a) Is it necessary to consider the ambiguous case? Explain.

Yes, because it is ambiguous so you need to find both solutions.

b) Sketch all possible diagrams for this situation.



c) Determine all possible the distances between Jason and Sammy to the nearest tenth of a meter.

$\frac{\sin 42}{12.9} = \frac{\sin B}{15.4}$

$\sin B = \frac{\sin 42 \times 15.4}{12.9}$

$= 0.7988...$

$\sin^{-1}(0.7988...)$

$\doteq 53.02$

$\frac{\sin 42}{12.9} = \frac{\sin 84.98}{SB}$

$SB = \frac{\sin 84.98 \times 12.9}{\sin 42}$

$\doteq 19.2$ — Solution ①

$\frac{\sin 42}{12.9} = \frac{\sin 11.02}{SB}$

$SB = \frac{\sin 11.02 \times 12.9}{\sin 42}$

$\doteq 3.69$ — solution ②