Ch1 Review

September 20, 2019 1:12 PM

Name:______________________ Date:____________________

Math 9 Honours Ch1 Review:

Rational & Irrational Numbers	Prime Factorizations
Square Roots and Mixed Radicals	Conversion with Decimals and Fractions
Factoring Trinomials	Divisibility Rules
BEDMAS with Irrational numbers	

Note: For the chapter test, please review all the assignments given through out the chapter. Calculators are not allowed

1. What are the properties of a rational number.

   ALL RATIONAL NUMBERS MUST BE ABLE TO BE REPRESENTED AS A FRACTION IN THE FORM a/b, WHERE BOTH NUMERATOR + DENOMINATOR ARE INTEGERS AND $b \neq 0$.

2. Provide 3 or more examples of an Irrational number

   ① $\pi = 3.1415\,91.....$
   ② $\sqrt{2}$
   ③ $e^1 = 2.718.....$

   ALL IRRATIONAL NUMBERS HAVE A DECIMAL REPRESENTATION THAT HAS NO CONSISTENT PATTERN AND IS NON-TERMINATING.

3. How do you determine that a value is a perfect square/cube from it's prime factorization

   ① ALL BASES NEED TO BE PRIME FACTORS
   ② ALL EXPONENTS MUST BE EVEN.

4. Evaluate the following without a calculator. Show your steps. Give your answer in exact form:

   i) $0.8 \div 0.\overline{1428\cancel{75}}^{\,75} - 0.375$

   $= \frac{4}{5} \div \frac{1}{7} - \frac{3}{8}$ $= \frac{28}{5} - \frac{3}{8}$

   $= \frac{4}{5} \times \frac{7}{1} - \frac{3}{8}$ $= \frac{196}{40} - \frac{15}{40}$

   28×7: 140, 56

   $= \frac{181}{40}$

   ii) $0.\overline{9} + 0.\overline{12} + 0.\overline{123}$

   $\frac{9}{9} + \frac{12}{99} + \frac{123}{999} = \frac{4{,}556}{3663}$

   $\frac{1}{1} + \frac{4}{33} + \frac{41}{333}$

   $\frac{3661}{3663} + \frac{444}{3663} + \frac{451}{3663}$

   iii) $0.\overline{1} \div 0.\overline{2} + 0.\overline{72} \times 0.375$

   $\frac{1}{9} \times \frac{9}{2} + \frac{72}{99} \times \frac{3}{8}$

   $\frac{1}{2} + \frac{8}{11} \times \frac{3}{8}$

   $\frac{1}{2} + \frac{3}{11} = \frac{11}{22} + \frac{6}{22} = \frac{17}{22}$

5. Given that 1/13 =0.076... and 2/13 = 0.153, then what is the 100th digit in the decimal expansion of 7/13?

   ① $\frac{1}{3} = 0.076\underline{9}\,\underline{2}\,\underline{3}$
   ② $\frac{2}{13} = 0.153\underline{8}\,\underline{4}\,\underline{6}$
   ③ $\frac{7}{13} = 0.\overline{615384}$
   ④ EVERY 6 TERMS WILL REPEAT.

   BEGINS WITH 6 B/C IT IS THE 6th LARGEST VALUE IN THESE TWO LISTS!

   $\frac{7}{13} = 0.615384$

   97th, 98th, 99th, 100th digit, 96th digit

6. Convert the following number to a fraction: i) $0.0\overline{773}$ ii) $0.023\overline{445}$

   i)
   ① $0.\overline{773} = \frac{773}{999}$
   ② $0.\overline{773} \longrightarrow 0.0\overline{773}$ DIVIDE BY 10 $\therefore \frac{773}{9990}$

   ii)
   ① $23.\overline{445} = 23\frac{445}{999}$
   ② $23.\overline{445} \div 1000 = 0.023\overline{445}$

   $23\frac{445}{999} \div 1000 = \frac{23{,}442}{999000}$

7. What is the sum of $0.00\overline{883}$ *and* $0.0\overline{325}$. Write your answer as a fraction in lowest terms.

   ① $0.\overline{883} = \frac{883}{999}$
   ② $0.\overline{883} \xrightarrow{\div 100.} 0.00\overline{883}$

   So $\frac{883}{999} \div 100 = \frac{883}{99900}$

   ③ $3.\overline{25} = 3\frac{25}{99}$

   $3.\overline{25} \xrightarrow{\div 100.} 0.0\overline{325}$

   $3\frac{25}{99} \div 100$

   $= \frac{322}{9900}$

   ④ $99900 = 999 \times 100$
   $= 9 \times 111 \times 100$

   $9900 = 99 \times 100$
   $= 9 \times 11 \times 100.$

   $\frac{883 \times 11 + 322 \times 111}{1098900} = \frac{45{,}455}{1098900}$

   $= \frac{9713 + 35742}{1098900} = \frac{9091}{219{,}780}$

8. Given that "x' is an ~~integer~~ RATIONAL NUMBER and $N=\sqrt{55-x^2}$. If "N" is a ~~rational number~~ AN INTEGER then how many possible values of "x" are there?

① $55-x^2=49$ $=1$
$=36$ $=0.$
$=25$
$=16$
$=9$
$=4$

← FOR EACH OF THESE VALUES, THERE ARE TWO OPTIONS FOR "x"

ie: if $55-x^2=49$ then $x^2=6$ $x=\pm\sqrt{6}$.

∴ THE TOTAL NUMBER OF 'x' IS $2\times 8=16$ POSSIBILITIES

9. Given that $N=\dfrac{\sqrt[4]{3^3\times 4^5\times 5^3\times 11^3}}{\sqrt{3^a\times 4^b\times 5^c\times 11^d}}$. If "N" is a rational number, then what is the lowest value of $a+b+c+d$?

① $a=1$ so $\sqrt{\frac{3^3}{3^1}}=\sqrt{3^2}=3$

② $b=0$ B/C THE BASE IS ALREADY A PERFECT SQ

③ $c=1$ ④ $d=1$.

④ $a+b+c+d$
$=3$

10. Arrange the following from least to greatest: $5\sqrt[3]{7}$, $\sqrt{33}$, $\sqrt[3]{200}$, $\sqrt[4]{1210}$, $2.5\sqrt{6}$
(5) (1) (2) (3) (4)

$=5\sqrt[3]{7}$
$=\sqrt[3]{125\times 7}$
$=\sqrt[3]{875}\approx 9.3$

$\sqrt[3]{729}<\sqrt[3]{875}<\sqrt[3]{1000}$
$\sqrt{25}<\sqrt{33}<\sqrt{36}$
$\sqrt[3]{125}<\sqrt[3]{200}<\sqrt[3]{216}$

$\sqrt[4]{625}<\sqrt[4]{1210}<\sqrt[4]{1296}$
$\sqrt{36}<2.5\sqrt{6}<\sqrt{49}$.

• $5\sqrt[3]{7}\approx 9.3$ • $\sqrt[4]{1210}\approx 5.95$
• $\sqrt{33}\approx 5.8$ • $2.5\sqrt{6}\approx 6.1$
• $\sqrt[3]{200}\approx 5.9$

11. Convert the following to mixed radicals: $\sqrt{10!}$

① $\sqrt{10!}=\sqrt{2^{\square}\times 3^{\square}\times 5^{\square}\times 7^{\square}}$

② $10\div 2=5$
$10\div 4=2$ ∴ 2^8
$10\div 8=1$

③ $10\div 3=3$
$10\div 9=1$ ∴ 3^4

④ $10\div 5=2$
$10\div 25=0$ ∴ 5^2

⑤ $10\div 7=1$ ∴ 7^1

So $\sqrt{10!}=\sqrt{2^8 3^4 5^2\times 7^1}$
$=2^4 3^2 5\sqrt{7}$

12. If $a\sqrt[3]{b}=\sqrt[3]{20!}$, then what is the lowest value of $a+b$?

① $20!=2^{\square}\times 3^{\square}\times 5^{\square}\times 7^{\square}\times 11^1\times 13^1\times 17^1\times 19^1$

② $20\div 2=10$
$20\div 4=5$
$20\div 8=3$
$20\div 16=1$
∴ 2^{19}

③ $20\div 3=6$
$20\div 9=2$
$20\div 27=0$
∴ 3^8

④ $20\div 5=4$
$20\div 25=0$
$20\div 7=2$

$\sqrt[3]{20!}=\sqrt[3]{2^{19}\times 3^8\times 5^4\times 7^2\times 11\times 13\times 17\times 19}$
$=2^6\cdot 3^2\cdot 5^1\sqrt[3]{2\times 3^2\times 5^1\times 7^2\times 11\times 13\times 17\times 19}$

$a=2^6\cdot 3^2\cdot 5^1$
$=2880$

$b=2\times 3^2\times 5\times 7^2\times 11\cdot 13\cdot 17\cdot 19$
$=263,693,490$

$a+b=263,696,370$

13. $\sqrt{a}\times\sqrt[3]{b}=\sqrt[p]{a^m\times b^n}$, what are the lowest possible values of "m" , "n", and "p"?

$\sqrt{a}=a^{1/2}$ $\sqrt[3]{b}=b^{1/3}$

∴ $a^{1/2}=a^{3/6}$ $b^{1/3}=b^{2/6}$.

So $a^{3/6}=\sqrt[6]{a^3}$ $b^{2/6}=\sqrt[6]{b^2}$.

So $\sqrt{a}\times\sqrt[3]{b}$
$=\sqrt[6]{a^3\times b^2}$

$m=3$ $b=2$ $p=6$

14. Given that $N=2^3\times 4^3\times 5^3\times 6^4$, how many factors does "N" have?

$N=2^3\times(2^2)^3\times 5^3\times 2^4\times 3^4$
$=2^3\times 2^6\times 2^4\times 3^4\times 5^3$
$=2^{13}\times 3^4\times 5^3$

So # of FACTORS IS
$=(13+1)(4+1)(3+1)$
$=14\times 5\times 4$
$=280$ FACTORS.

15. Using the value of "N" from above, how many factors does "N" have that are perfect squares?

$N = 2^{13} \times 3^4 \times 5^3$ # of FACTORS THAT ARE PERFECT SQUARES.

$= \begin{matrix} 2^0 & 3^0 & 5^0 \\ 2^2 & 3^2 & 5^2 \\ 2^4 & \times 3^4 & \times \\ 2^6 & & \\ 2^8 & & \\ 2^{10} & & \\ 2^{12} & & \end{matrix} = 7 \times 3 \times 2$

= 42 FACTORS ARE PERFECT SQUARES.

16. Given that $N = (x^2 - 13x + 36)(x+3)$ and "N" is a perfect square. What are all the possible values of "x"?

① FACTOR $x^2 - 13x + 36$
$= (x-4)(x-9)$

② $N = (x-4)(x-9)(x+3)$
$= A \times B \times C$

③ FOR "N" TO BE A PERF SQ,
• $AB = C$ or $AC = B$ or $BC = A$.

$x^2 - 13x + 36 = x + 3$
$x^2 - 14x + 33 = 0$
$(x-3)(x-11) = 0$
$x = 3, x = 11$

$x^2 - x - 12 = x - 9$
$x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$
$x = 3, x = -1$

$x^2 - 6x - 27 = x - 4$
$x^2 - 7x - 23 = 0$ [QUADR. FORM]
$x = \frac{7 \pm \sqrt{49 + 4(23)}}{2}$
$= \frac{7 \pm \sqrt{141}}{2}$

$92 + 49 = 141$

17. Given that $N = 2(x-1)(x+2)(3x-1)$ and "N" is a perfect square. What are all the possible values of "x"?

let $N = 2(A) \times B \times C$.

① $2A = BC$ ② $A = 2BC$ ③ $2B = AC$ ④ $B = 2AC$ ⑤ $2C = AB$ ⑥ $C = 2AB$

① $2(x-1) = (x+2)(3x-1)$
$2x - 2 = 3x^2 + 6x - x - 2$
$0 = 3x^2 + 3x$
$0 = 3x(x+1)$
$x = 0 \quad x = -1$

② $x - 1 = 2(x+2)(3x-1)$
$x - 1 = 6x^2 + 10x - 4$
$0 = 6x^2 + 9x - 3$
$0 = 2x^2 + 3x - 1$
$x = \frac{-3 \pm \sqrt{9 + 4(2)}}{4}$
$x = (-3 \pm \sqrt{17})/4$

③ $2(x+2) = (x-1)(3x-1)$
$2x + 4 = 3x^2 - 4x + 1$
$0 = 3x^2 - 6x - 3$
$0 = x^2 - 2x - 1$
$x = \frac{2 \pm \sqrt{4 + 4}}{2}$
$x = (2 \pm 2\sqrt{2})/2 = 1 \pm \sqrt{2}$

④ $(x+2) = 6x^2 - 8x + 2$
$0 = 6x^2 - 9x$
$0 = 3x(2x - 3)$
$x = 0 \quad x = 3/2$

⑤ $2(3x-1) = (x-1)(x+2)$
$6x - 2 = x^2 + x - 2$
$0 = x^2 - 5x$
$0 = x(x-5)$
$x = 0 \quad x = 5$

⑥ $3x - 1 = 2(x-1)(x+2)$
$3x - 1 = 2x^2 + 2x - 4$
$0 = 2x^2 - x - 3$
$0 = (2x-3)(x+1)$
$x = 3/2 \quad x = -1$

18. When given the prime factorization with a missing term, what is the value of the term required to create a perfect square/cube

a) Given that "N" is an integer, what is the lowest value of "k" if "k" is an positive integer?

$N = \sqrt{32k}$

$N = \sqrt{2^5 \times k}$

$\therefore k = 2.$

$N = \sqrt{3^3 6^3 7^1 (k+1)}$

$N = \sqrt{3^3 \times 2^3 \times 3^3 \times 7 \times (k+1)}$
$= \sqrt{3^6 \times 2^3 \times 7^1 (k+1)}$
$\therefore k + 1 = 2 \times 7$
$\boxed{k = 15}$

$N = \sqrt[3]{5^2 7^3 (k-1)}$

$k - 1 = 5$
$\boxed{k = 6}$

b) Challenge: Given that "N" is a perfect square, what is the lowest integer value of "k" if $k \geq 1$. (Note: "N" needs to be a perfect square, not an integer)

$N = \sqrt{32k}$

$N = \sqrt{32 \times k}$
$N = \sqrt{2^5 \times k}$ $k = 8.$
$N = \sqrt{2^8} = 2^4$

• EXPONENTS NEED TO BE A POWER OF 4

$N = \sqrt{3^3 6^3 7^1 (k+1)}$

$N = \sqrt{3^3 \times 2^3 \times 3^3 \times 7 \times (k+1)}$
$N = \sqrt{3^6 \times 2^3 \times 7 \times (k+1)}$
$k + 1 = 3^2 \times 2 \times 7^3.$
$N = \sqrt{3^8 \times 2^4 \times 7^4} = 3^4 \times 2^2 \times 7^2.$

c) what is the lowest integer value of "k" if $k \geq 1$, such that "N" is a perfect cube

$$N = (30k+5)(15k+4)$$

① SINCE WE ARE LOOKING FOR THE "LOWEST POSITIVE INTEGER, THEN JUST USE TRIAL & ERROR, START FROM 1,"

i) $N = A^3 \times B^3$. So Trial & error.

	30k+5	15k+4	N
k=1	35	19	x
k=2	65	34	x
k=3	95	49	x
k=4	125	64	✓

19. Prove that sum of $1+3+5+.....+13+15+17+.....(2n+1)$ will always be a perfect square. Note: "n" is a natural number.

① · THIS SEQUENCE IS ARITHMETIC

$$\text{SUM} = \frac{\text{\# OF TERMS}}{2} \times \left(\text{FIRST + LAST TERMS}\right)$$

② # of TERMS = n+1

③ FIRST TERM = 1
LAST TERM = 2n+1.

④ $\text{Sum} = \frac{(n+1)}{2}\left[1 + 2n+1\right]$

$= (n+1)\frac{(2n+2)}{2} = (n+1)(n+1)$

$= (n+1)^2$ //

20. How many positive integers less than 1000 is equal to the product of three different primes?

21. The product of N consecutive four-digit positive integers is divisible by 2010^2. What is the least possible value of N?

(A) 5 (B) 12 (C) 10 (D) 6 (E) 7

① $2010^2 = (201 \times 10)^2$

$= (3 \times 67 \times 2 \times 5)^2$

$= 2^2 \times 3^2 \times 5^2 \times 67^2$ //

② USE 67^2 TO MAKE ONE OF THE 4 DIGIT #'S //

$67^2 = 4489$.

4485 | 4486 | 4487 | 4488 | 4489 | 4490

YOU NEED THESE TWO TERMS TO MAKE TWO 5'S

③ ANOTHER POSSIBILITY IS USE $67^2 \times 2$ TO MAKE THE 4 DIGIT #.

8975 8976 8977 8978 8979.

THIS TERM WILL CONTAIN TWO 5'S.

∴ N = 5 CONSECUTIVE TERMS.

22. If $x^2yz^3 = 7^4$ and $xy^2 = 7^5$, then xyz equals

(A) 7 **(B)** 7^2 **(C)** 7^3 **(D)** 7^8 **(E)** 7^9

$(x^2 y \cdot z^3)(xy^2) = 7^4 \times 7^5$

$(x^3 \cdot y^3 \cdot z^3)^{1/3} = (7^9)^{1/3}$

$xyz = 7^3$

$= 343$ //

23. Solve problems involving divisibility rules:

a) Given that $N = 2389b$ and is divisible by 12. What is the lowest value for "b"

① LAST TWO DIGITS FORM A NUMBER DIVISIBLE BY 4.

$92 \rightarrow b = 2$

$96 \rightarrow b = 6.$

NOTE: IF A NUMBER IS DIVISIBLE BY BOTH 3 & 4, IT MUST THEN BE DIVISIBLE BY 12//

② CHECK WHICH VALUE OF 'b' WILL BE DIVISIBLE BY 3.

$23892 \div 3 = 7964$

$23896 \div 3 = 7965.\overline{33}$

$\therefore b = 2$ //

22. A five-digit positive integer is created using each of the odd digits 1, 3, 5, 7, 9 once so that

- the thousands digit is larger than the hundreds digit, D > C
- the thousands digit is larger than the ten thousands digit, D > E
- the tens digit is larger than the hundreds digit, and B > C
- the tens digit is larger than the units digit. B > A.

Find as many number that satisfy these properties:

① E D C B A

THOUSANDS TENS

② D & B CAN ONLY BE 7 or 9 INTERCHANGEABLY.

③ E C A CAN BE 1, 3, or 5 INTERCHANGEABLY

④ DB → 2 ways 79 or 97

ECA → 6 ways (3!) or 135 513 153 531 351 315

$\therefore 2 \times 6 = 12$ DIFFERENT NUMBERS//

23. Find a number that contains all the digits from 0 to 9 (repetitions allowed) such that when you multiply it with any number from 1 to 18, the product will also contain digits from 0 to 9 (repetitions allowed and can be in any order). Hint the number is a big value: