

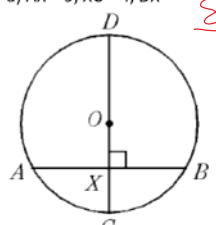
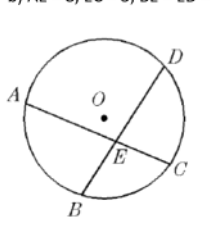
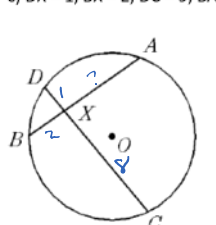
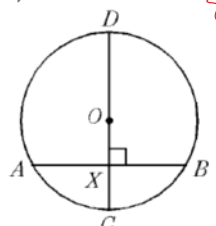
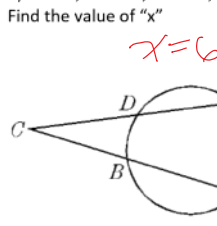
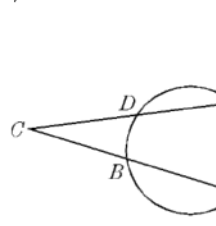
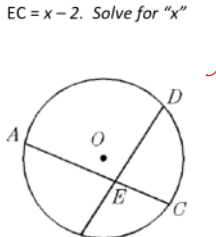
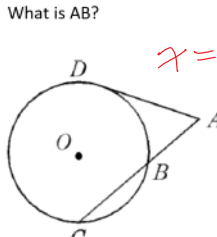
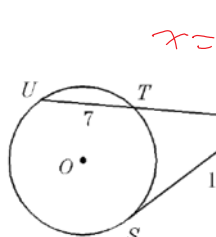
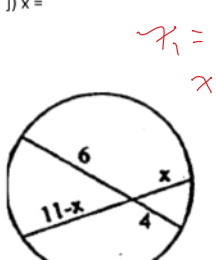
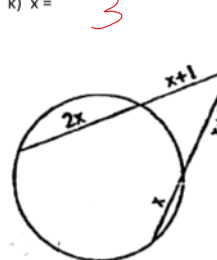
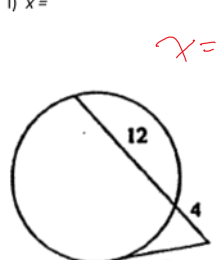
Name: \_\_\_\_\_

*Koyan*

Date: \_\_\_\_\_

**Math 9 Enriched: Assignment 7.5 Intersecting Chords, Secants, and Tangents in Circles**

1. Find the length of the missing sides for each of the following:

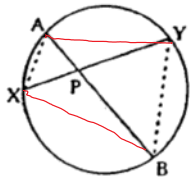
|   |   |  |
|---|---|--|
| <p>a) <math>AX = 9, XC = 4, DX =</math> <i><math>\frac{81}{4}</math></i></p>   | <p>b) <math>AE = 8, EC = 3, BE = ED = ?</math> <i><math>2\sqrt{6}</math></i></p>                           | <p>c) <math>DX = 1, BX = 2, DC = 9, BA =</math> <i><math>4</math></i></p>  |
| <p>d) <math>OX = CX = 4\text{cm. } AB =</math> <i><math>8\sqrt{3}</math></i></p>                                       | <p>e) <math>CD = 8, DE = 12, CB = 10, BA = x</math><br/>Find the value of "x" <i><math>x=6</math></i></p>  | <p>f)</p>    |
| <p>g) <math>AE = x + 4, ED = x, BE = x - 1, EC = x - 2.</math> Solve for "x" <i><math>x = \frac{8}{3}</math></i></p>  | <p>h) AD is a tangent, <math>AD = 6, AC = 9,</math><br/>What is AB? <i><math>x=4</math></i></p>           | <p>i) <math>TR = ?</math> <i><math>x=9</math></i></p>                     |
| <p>j) <math>x =</math> <i><math>x_1 = 3, x_2 = 8</math></i></p>    | <p>k) <math>x =</math> <i><math>3</math></i></p>   | <p>l) <math>x =</math> <i><math>x=8</math></i></p>                       |

*$12 \times 12$   
 $9 \times 16$*

*$12^2 = x(x+7)$   
 $0 = x^2 + 7x - 144$   
 $0 = (x-9)(x+16)$   
 $x=9, x=-16$*

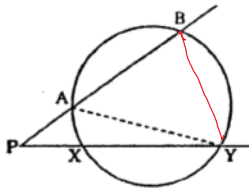
*$x^2 = 4(16)$   
 $x=8$*

2. Prove the following equation:  $PA \cdot PB = PX \cdot PY$ .



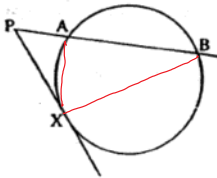
$$\begin{aligned} \angle APX &= \angle YPB & \angle AYX &= \angle APX \\ \angle YAB &= \angle YXB \\ \Delta APX &\sim \Delta BPY \\ \frac{AP}{PY} &= \frac{PX}{PB} & \therefore PA \cdot PB &= PY \cdot PX \end{aligned}$$

3. Prove the following equation:  $PA \cdot PB = PX \cdot PY$



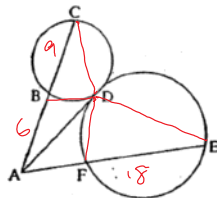
$$\begin{aligned} \angle PAX &= \angle BYP & \Delta PAX &\sim \Delta PYB \\ \frac{PA}{PX} &= \frac{PY}{PB} & PA \cdot PB &= PX \cdot PY \end{aligned}$$

4. Prove the following equation:  $PA \cdot PB = (PX)^2$



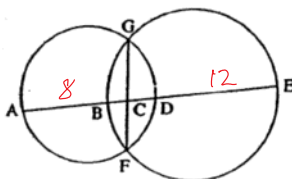
$$\begin{aligned} \angle AXP &= \angle PBX \\ \Delta PAX &\sim \Delta PXB \\ \frac{PA}{PX} &= \frac{PX}{PB} & PA \cdot PB &= (PX)^2 \end{aligned}$$

5.  $AB = 6$ ,  $BC = 9$ , and  $AE = 18$ . Find  $AF$ :



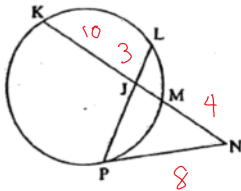
$$\begin{aligned} (AB)(AC) &= (AD)^2 & AD &= 3\sqrt{10} \\ (AF)(AE) &= (AD)^2 & AF &= 5 \end{aligned}$$

6.  $AB = 8$ ,  $BD = 7$ , and  $DE = 12$ . Find the length of  $BC$ .



$$\begin{aligned} BC &= x & CD &= 7-x \\ (AC)(CD) &= (AG)(GF) = (BC)(CE) \\ (8+x)(7-x) &= x(19-x) \\ 56 + x - x^2 &= 19x - x^2 \\ x &= \frac{28}{9} \end{aligned}$$

7. Given that  $JK = 10$ ,  $JL = 3$ ,  $MN = 4$ , and  $PN = 8$ . Find the length of  $JP$ .

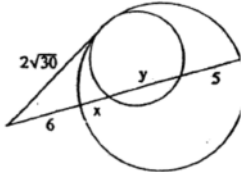


$$JP = x \quad JM = y$$

$$(14+y)4 = 8^2 \quad y = 2$$

$$(KJ)(JM) = (LN)(JP) \quad JP = \frac{20}{3}$$

8. Find the length of " $x + y$ ".

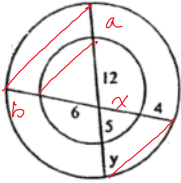


$$(2\sqrt{30})^2 = 6(11+x+y)$$

$$120 = 66 + 6(x+y)$$

$$x+y = 9$$

9. For the two concentric circles shown, find the value of " $y$ " to 2 decimal places.

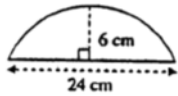


$$12 - 5 = 6 - x \quad x = 10$$

$$\frac{x+4}{12} = \frac{y+5}{6}$$

$$y = 2$$

10. Part of a circular plate has the measurements given on the diagram. Show two or more ways to calculate the radius of the plate.



$$12^2 = 6(r+r-6)$$

$$144 = 12r - 36$$

$$r = 15$$

$$r^2 = (r-6)^2 + 12^2$$

$$r^2 = r^2 - 12r + 36 + 144$$

$$r = 15$$

11. In the diagram,  $C$  is the centre of the circle and  $AD$  is tangent to the circle at  $D$ .  $AC$  is a straight line. If  $AD = 10$ , and  $AB = 7$ , what is the length of " $BC$ "?

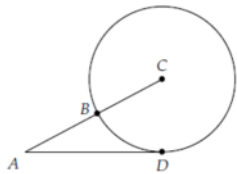
a)  $\frac{\sqrt{151}-7}{2}$

b)  $\sqrt{14}$

c)  $\frac{51}{14}$

d)  $\frac{\sqrt{51}}{2}$

e)  $\frac{7}{2}$



$$BC = CD = x$$

$$(7+x)^2 = 10^2 + x^2$$

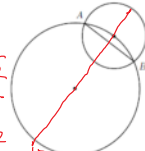
$$x^2 + 14x + 49 = x^2 + 100$$

$$x = \frac{51}{14}$$

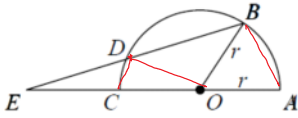
12. Two circles are shown in the diagram. The larger circle has radius 2, and the smaller circle has radius 1 and its centre is on the circumference of the larger circle. What is the length of chord AB?

- a)  $\frac{\sqrt{15}}{2}$     b)  $\frac{5\sqrt{34}}{17}$     c) 2    d) 1.5    e)  $\frac{15}{8}$

$O = P = x$ ;  $PO = 1 - x$      $\frac{AB}{2} = y$      $y = \frac{\sqrt{15}}{2}$   
 $(4 - x)x = y^2 - (1 - x)(1 + x)$ ;  $4x - x^2 = 1 - x^2$ ;  $x = \frac{1}{4}$



13. In the diagram, O is the centre of the circle with radius "r". ED = "r" and  $\angle DEC = k \times \angle BOA$ . What is the value of "k"?

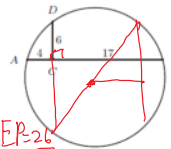


$BOB = x$ ;  $\angle OBO = x$ ;  $\angle BOD = 180 - 2x$   
 $\angle EDO = 180 - x$ ;  $\angle DEO = \angle DOE = \frac{x}{2}$   
 $\angle BOA = \frac{3}{2}x$      $\angle DEC = \frac{1}{3} \angle BOA$      $k = \frac{1}{3}$

14. In the circle shown, line segment CD is perpendicular to the chord AB. Further AC = 4, CD = 6, and BC = 17. What is the radius of the circle?

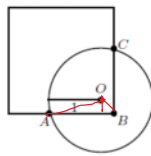
- a)  $\frac{65}{6}$     b) 11    c)  $\frac{21}{2}$     d)  $\frac{\sqrt{513}}{2}$     e) 13

$(DC)(CF) = (AC)(BC)$ ;  $CF = \frac{34}{3}$   
 $OP = \frac{21}{2} \cdot 4 = \frac{13}{2}$ ;  $OE = \frac{65}{6}$



15. A circle of radius 1 has centre near one vertex of a square in such a way that  $AB = BC = "a"$ . What is the value of "a" for which the distance OB is equal to  $\frac{1}{2}$ ?

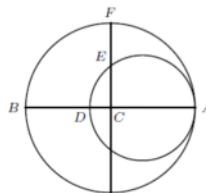
- (A)  $\frac{\sqrt{3}-1}{2}$     (B)  $\frac{\sqrt{7}+1}{2\sqrt{2}}$     (C)  $\frac{\sqrt{3}+1}{2}$   
 (D)  $\frac{\sqrt{7}+1}{\sqrt{2}}$     (E)  $\frac{\sqrt{7}-1}{2\sqrt{2}}$



$AB = CB$ ;  $\angle DBA = 45^\circ$   
 $OP \perp AB$ ;  $OP = PB = \frac{1}{\sqrt{2}}$   
 $OA = 1$ ;  $AP = \frac{\sqrt{2}}{2}$

16. Two circles are tangent to each other at "A" and the centre of the larger circle is at "C". The lines AB and FC are perpendicular diameters of the larger circle. If BD = 9cm and FE = 5cm, what is the radius of the smaller circle?

- (a) 14    (b) 18    (c)  $19\frac{1}{2}$   
 (d)  $20\frac{1}{2}$     (e) 21



$AC = R$ ;  $R(R - 9) = (R - 5)^2$   
 $R^2 - 9R = R^2 - 10R + 25$   
 $R = 25$   
 $r = \frac{25 + (25 - 9)}{2}$      $r = \frac{41}{2}$      $(20\frac{1}{2})$