Ch 2 Review

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Name:____________________________ Date:______________________

Math 10 Honors Chapter 2 Review: Sequences and Series:

1. Find the sum of the following sequences:
   a. $8+5+2+....+(-10)$
      $a=8$ $d=-3$ $t_n=-10=8+(n-1)(-3)$ $-18=(n-1)(-3)$ $6=n-1$ $\boxed{7=n}$
      $S_7=\frac{7}{2}(8+(-10))$ $\boxed{S_7=7}$
   b. (First 100 terms) $(-101)+(-99)+(-97)+.....$
      $a=-101$ $d=2$ $n=100$
      $S_{100}=\frac{100}{2}[2(-101)+(100-1)(2)]$ $=50[-202+198]$ $=50(-4)=-200$
   c. Find the sum of the first "k" integers: $1+2+3+....+k$
      $\boxed{=\frac{k(k+1)}{2}}$
   d. Find the sum of the first forty terms: $(-59)+(-56)+(-53)+.....$
      $a=-59$ $d=3$ $n=40$
      $S_{40}=\frac{40}{2}[2(-59)+(40-1)3]$ $=20[-118+117]$ $=20(-1)=-20$
   e. Find the sum to infinity: $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+.....$
      ① $\frac{1}{7}+\frac{1}{7^3}+\frac{1}{7^5}+.....=\frac{\frac{1}{7}}{1-\frac{1}{7^2}}=\frac{7}{48}$
      ② $\frac{2}{7^2}+\frac{2}{7^4}+\frac{2}{7^6}+.....=2\left[\frac{\frac{1}{7^2}}{1-\frac{1}{7^2}}\right]=2\left(\frac{1}{48}\right)=\frac{1}{24}$
      ③ $\frac{7}{48}+\frac{2}{48}$ $=\frac{9}{48}$

2. If the sum of the first ten terms of an arithmetic sequence is four times the sum of the first five terms, find the ratio of the first term to the common difference.
   $S_{10}=10a+45d$ $S_5=5a+10d$
   $10a+45d=4(5a+20d)$
   $\cancel{10a}+45d=\cancel{20a}+80d$
   $-35d=10a$
   $-7d=2a$
   RATIO: $a:d$ or -3.5
   $-7:2$

3. What is the sum of the series: $1+2+4+.....+2^k$? $=\frac{2^k-1}{2-1}$

4. Prove the following equation: $x^k-1=(x-1)(x^{k-1}+x^{k-2}+....+x+1)$
   $(x-1)\left[\frac{x^k-1}{x-1}\right]=x^k-1$

5. What is the value of $\sum_{k=1}^{4}\left(\sum_{j=1}^{4}kj\right)$? $\sum_{k=1}^{4}\left(j(1+2+3+4)\right)$
   $(1+2+3+4)(1+2+3+4)$
   $=100$

6. What is the sixth term of the arithmetic sequence whose 31st and 73rd terms are 18 and 46, respectively?
   ① $t_{31}=a+30d=18$ ② $t_{73}=a+72d=46$
   $t_{73}-t_{31}=42d=28$
   $\boxed{d=\frac{2}{3}}$
   $a+30(\frac{2}{3})=18$
   $a+20=18$
   $\boxed{a=-2}$
   $t_6=-2+(5)(\frac{2}{3})$ $=-2+\frac{10}{3}$ $=-\frac{6}{3}+\frac{10}{3}=\frac{4}{3}$

7. The second term of a geometric sequence is 4 and the sixth term is 16. Find the fourth term if the ratio of consecutive terms is a real number.
   ① $t_2=ar=4$
   $t_6=ar^5=16$
   $r^4=4$
   $\boxed{r=\pm\sqrt{2}}$
   ② $\frac{}{t_1}$, $\frac{4}{t_2}$, $\frac{\pm4\sqrt{2}}{t_3}$, $\boxed{\frac{8}{t_4}}$, $\pm8\sqrt{2}$, 16
   $t_4=8$

8. For what values of "x" is the equation true? $1+x+x^2+x^3+x^4+.....=4$?

$a=1 \quad r=x$ $\quad \frac{1}{1-x}=4$ $\quad \frac{1}{4}=1-x$ $\quad x=\frac{3}{4}$

9. If five geometric means are inserted between 8 and 5832, find the fifth term in the geometric sequence formed by the seven numbers.

① 8 _ _ _ _ _ 5832

② $5832 = 8 \times r^6$

$729 = r^6$

$\pm 3 = r$

③ $8, \pm 24, 72, \pm 216, 648,$

$t_5 = 648$

10. Find "x" so that the sequence: $4x-1,\ 2x+2,\ 2x-3$ is an arithmetic progression.

$t_3 - t_2 = t_2 - t_1$

$(2x-3)-(2x+2) = (2x+2)-(4x-1)$

$-5 = -2x+3$

$2x = 8$

$\boxed{x=4}$

Check: $15, 10, 5$ (-5, -5)

11. If x, m, n, y is an arithmetic sequence and x, a, b, c, y is another arithmetic sequence, which of the following must be true? $i)\ m+n=a+c \quad ii)\ n-m=c-b \quad iii)\ a+b+c=2(m+n)$

① $y-n=n-m$ $\quad \boxed{y+m=2n}$ $\quad n-m=m-x$ $\quad \boxed{n+x=2m}$

$n+x=2m$ $\quad \boxed{y+x=m+n}$

$y-c=c-b$ $\quad \boxed{y+b=2c}$ $\quad b-a=a-x$ $\quad \boxed{b+x=2a}$

$b+x=2a$ $\quad y+x+2b=2c+2a$

$c-b=b-a$ $\quad c+a=2b$

$m+n+(2b)=2c+2a$

$m+n+(a+c)=2c+2a$

$\boxed{m+n=a+c}$ #1)

12. If $a, b, c, ...$ is an arithmetic sequence, which of the following is also arithmetic?

$i)\ 5a+4,\ 5b+4,\ 5c+4, ...$

$ii)\ \frac{a}{2}, \frac{b}{2}, \frac{c}{2}$

$iii)\ a+3,\ b-1,\ c-5$

① plug $a=2,\ b=4,\ c=6$

$5a+4=14 \quad 5b+4=24 \quad 5c+4=34$ YES

② $\frac{a}{2}=1 \quad \frac{b}{2}=2 \quad \frac{c}{2}=3$ YES

③ $a+3=5,\ b-1=3,\ c-5=1$ YES

13. For what value(s) of "x" will $4, x+7, x^2+11$ form a geometric sequence?

$\frac{x^2+11}{x+7} = \frac{x+7}{4}$

$4x^2+44 = x^2+14x+49$

$3x^2-14x-5=0$

$(3x+1)(x-5)=0$

$x=-\frac{1}{3} \quad x=5$

14. Find the sum of all the multiples of 3 from 200 to 299:

$201 = 3(67)$

$297 = 3(99)$

$\therefore S = 3(67)+3(68)+....3(99)$

$= 3(67+......99) = \frac{99}{2}[166]$

$= 3\left[\frac{33}{2}(67+99)\right] = 99 \times 83 = 8217$

$8300-83$

8217

15. Terms a, b, c, d, e form an arithmetic sequence. If $b+e=39$ & $a+c+d=46$, find the common difference.

a (a), $a+d$ (b), $a+2d$ (c), $a+3d$ (d), $a+4d$ (e).

$2a+5d=39$

$3a+5d=46$

$2a+5d=39$

$a=7$

$\therefore 14+5d=39$

$5d=25$

$\boxed{d=5}$

16. Given that a, x_1, x_2, b and $y_1,\ a,\ y_2,\ b,\ y_3$ are two arithmetic sequences, find the value of $\frac{x_2-x_1}{y_3-y_1}$

① $a \quad x_1 \quad x_2 \quad b$

$a,\ a+d,\ a+2d,\ a+3d.$

$b-a=3d.$

② $y_1,\ a,\ y_2,\ b,\ y_3.$

$a-m,\ a,\ a+m,\ a+2m,\ a+3m$

$b-a=2m.$

$\therefore \boxed{3d=2m}$

$\boxed{d=\frac{2m}{3}}$

③ $x_2-x_1=d$

$y_3-y_1=4m.$

So $\frac{x_2-x_1}{y_3-y_1} = \frac{\left(\frac{2m}{3}\right)}{(4m)}$

$= \frac{2}{12} = \frac{1}{6}$