CSMC and CIMC Prep #7)

Question #1) Solve for “x”: $\sqrt[3]{(2+x)^2}+3\sqrt[3]{(2-x)^2}=4\sqrt[3]{4-x^2}$ (CSMC 2014)

Question #2)

The integer 43797 satisfies the following conditions:

• each pair of neighbouring digits (read from left to right) forms a two-digit prime number, and

• all of the prime numbers formed by these pairs are different, because 43, 37, 79, and 97 are all different prime numbers.

There are many integers with more than five digits that satisfy both of these conditions. What is the largest positive integer that satisfies both of these conditions?

Question #3) If *"p"* and *"q"* are positive integers, *max(p, q)* is the maximum of *"p"* and *"q"* and *min(p, q)* is the minimum of *"p"* and *"q"*. For example, *max(30, 40) = 40* and *min(30, 40) = 30*. Also, max(30, 30) = 30 and min(30, 30) = 30.

Determine the number of ordered pairs (x,y) that satisfy the equation:

$\max\left(60, \min\left(x, y\right)\right) = \min\left(\max\left(60, x\right), y\right)$, where "x" and "y" are positive integers between 0 and 100. (CSMC 2013)

Question #4) Lynne is tiling her long and narrow rectangular front hall. The hall is 2 tiles wide and 13 tiles long. She is going to use exactly 11 black tiles and exactly 15 white tiles. Determine the number of distinct ways of tiling the hall so that no two black tiles are adjacent (that is, share an edge). (CSMC 2012)

Question #5) Given the systems of equations ,
$$\begin{aligned} x+y+z &= 1 \\ x^2+y^2+z^2 &= 3 \\ x^3+y^3+z^3 &= 7 \end{aligned}$$
, what is the value of $x^5+y^5+z^5$? (Brilliant L3)

Answers:

1) X=13/7, x=0
2) N = 619737131179.
3) 4100 pairs
4) 486 possible ways

Let $S_1 = x+y+z$, $S_2 = xy+xz+yz$, $S_3 = xyz$,

$P_n = x^n + y^n + z^n$.

According to Newton's Identities, we have

①. $P_1 = S_1 = 1$;

②. $P_2 = S_1P_1 - 2S_2 \Rightarrow 3 = 1 - 2S_2 \Rightarrow S_2 = -1$;

③. $P_3 = S_1P_2 - S_2P_1 + 3S_3 \Rightarrow 7 = 3+1+3S_3 \Rightarrow S_3 = 1$;

④. $P_4 = S_1P_3 - S_2P_2 + S_3P_1 = 7+3+1 = 11$;

⑤. $P_5 = S_1P_4 - S_2P_3 + S_3P_2 = 11+7+3 = \boxed{21}$.