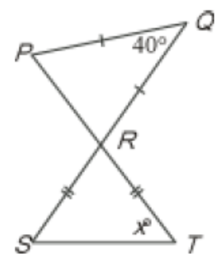
1. Determine the value of $\dfrac{\sqrt{25-16}}{\sqrt{25}-\sqrt{16}}$.

{2008 Cayley #2}

2. In the diagram, PT and QS are straight lines intersecting at R such that $QP = QR$ and $RS = RT$. Determine the value of x.



{2008 Cayley #8}

3. If $x + y + z = 25$, $x + y = 19$ and $y + z = 18$, determine the value of y.

{1998 Cayley #11}

4. The odd numbers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9 and 17 are placed as shown, what is the value of x?

	5	
9		17
x		

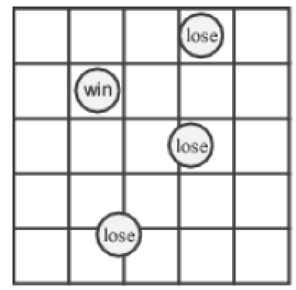
{2010 Cayley #16}

5. What is the largest positive integer n that satisfies $n^{200} < 3^{500}$?

{2010 Cayley #20}

6. A coin that is 8 cm in diameter is tossed onto a 5 by 5 grid of squares each having side length 10 cm. A coin is in a winning position if no part of it touches or crosses a grid line, otherwise it is in a losing position. Given that the coin lands in a random position so that no part of it is off the grid, what is the probability that it is in a winning position?

{2010 Cayley #24}



7. Daryl first writes the perfect squares as a sequence

$$1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \ldots$$

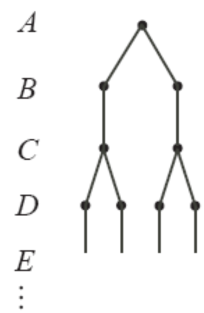
After the number 1, he then alternates by making two terms negative followed by leaving two terms positive. Daryl's new sequence is

$$1, -4, -9, 16, 25, -36, -49, 64, 81, -100, \ldots$$

What is the sum of the first 2011 terms in this new sequence?

{2011 Gauss 8 #25}

8. In the diagram, there are 26 levels, labelled $A, B, C, \ldots, Z$. There is one dot on level A. Each of levels $B, D, F, H, J, \ldots$, and Z contains twice as many dots as the level immediately above. Each of levels $C, E, G, I, K, \ldots$, and Y contains the same number of dots as the level immediately above. How many dots does level Z contain?



{2011 Pascal #21}

9. An ordered list of four numbers is called a *quadruple*.
A quadruple (p, q, r, s) of integers with $p, q, r, s \geq 0$ is chosen at random such that

$$2p + q + r + s = 4$$

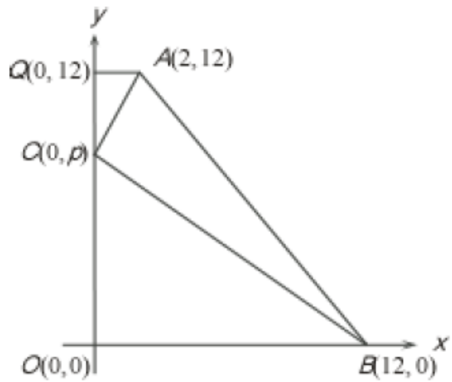
What is the probability that $p + q + r + s = 3$?

{2011 Pascal #23}

10. Let n be the largest integer for which $14n$ has exactly 100 digits. Counting from right to left, what is the 68th digit of n?

{2011 Pascal #24}

(b) Point $C(0,p)$ lies on the y-axis between $Q(0,12)$ and $O(0,0)$ as shown. Determine an expression for the area of $\triangle COB$ in terms of p.



Solution

13. If m is a positive integer, the symbol $m!$ is used to represent the product of the integers from 1 to m. That is, $m! = m(m-1)(m-2)\dots(3)(2)(1)$. For example, $5! = 5(4)(3)(2)(1)$ or $5! = 120$. Some positive integers can be written in the form

$$n = a(1!) + b(2!) + c(3!) + d(4!) + e(5!).$$

In addition, each of the following conditions is satisfied:

- $a, b, c, d,$ and e are integers
- $0 \le a \le 1$
- $0 \le b \le 2$
- $0 \le c \le 3$
- $0 \le d \le 4$
- $0 \le e \le 5.$

(a) Determine the largest positive value of N that can be written in this form.

(b) Write $n = 653$ in this form.

(c) Prove that all integers n, where $0 \le n \le N$, can be written in this form.

(d) Determine the sum of all integers n that can be written in this form with $c = 0$.