Name:______________________ Date:_____________________

Math 9 Enriched: Section 4.5 Factoring Difference and Sums of Powers

Difference of squares: $a^2-b^2=(a+b)(a-b)$

Difference and Sums of Cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$

Difference of Powers: $a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+...+ab^{n-2}+b^{n-1})$

Difference of Sums: $a^{2n+1}+b^{2n+1}=(a+b)(a^{2n}-a^{2n-1}b+a^{2n-2}b^2...-ab^{2n-1}+b^{2n})$

1. Factor each and simplify the following expressions completely:

a) x^6-64	b) $9^3-a^6x^6$
c) $81-(3a+2)^4$	d) $\dfrac{1000+27x^3}{100-9x^2}$
e) $\dfrac{a^3-27b^3}{a^2-9b^2}$	f) y^6+16y^3+15
g) x^6-7x^3-8	h) $8y^6-9y^3+1$
i) x^6-26x^3-27	j) $27y^6+35y^3+8$

2. Factor completely: $-a^2b^2+2ab^3-b^4+a^2c^2-2abc^2+b^2c^2$

3. Factor completely with integeral coefficients: $x^{12}-y^{12}$

4. Factor and simplify the expression as much as possible: $\left(\dfrac{a^3-1}{a^2-1}\right)\left(\dfrac{a^2+2a+1}{a^3+1}\right)\left(\dfrac{a^2-a+1}{a+1}\right)$

5. When x^9-x is factored as completely as possible into polynomials and monomials with integral coefficients, how many factors are there?

6. If $x+y=4$ and $xy=2$, then find x^6+y^6

7. Find the value of $x^6+\dfrac{1}{x^6}$ if the value of $x+\dfrac{1}{x}=3$.

8. If $a+b=1,\ a^2+b^2=2$, find the value of a^4+b^4

9. If $\frac{1}{a+c}=\frac{1}{a}+\frac{1}{c}$, find the value of $\left(\frac{a}{c}\right)^3$.

10. Find the sum of $\frac{1}{3+2\sqrt{2}}+\frac{1}{2\sqrt{2}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}+2}+\frac{1}{2+\sqrt{3}}$.

11. Challenge: Find the sum of: $\frac{1}{\sqrt[3]{1}+\sqrt[3]{2}+\sqrt[3]{4}}+\frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}+\frac{1}{\sqrt[3]{9}+\sqrt[3]{12}+\sqrt[3]{16}}$

12. Challenge: March 2009 (Adler). Show that $n^{n-1}-1$ is divisible by $(n-1)^2$ for every positive integer "n".

13. (Challenge) Let "x" and "y" be two-digit integers such that "y" is obtained by reversing the digits of "x". The integers "x" and "y" satisfy $x^2-y^2=m^2$ for some positive integer "m". What is the value of $x+y+m$?