Name:_______________________ Date:______________________

Section 4.3 Factoring Difference of Squares and Cubes

Difference of squares: $a^2 - b^2 = (a+b)(a-b)$

Difference and Sums of Cubes: $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Difference of Powers: $x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + ... + xy^{n-2} + y^{n-1})$

Difference of Sums: $x^{2n+1} + y^{2n+1} = (x+y)(x^{2n} - x^{2n-1}y + x^{2n-2}y^2 ... - xy^{2n-1} + y^{2n})$

1. Factor each of the following expressions:

a) $16x^2 - 49y^2$	b) $(2x-1)^2 - 9x^2$	c) $81a^2 - (3a+2b)^2$
d) $4(2x-y)^2 - 25z^2$	e) $18x^2y^2 - 50y^4$	f) $\frac{x^2}{16} - \frac{y^2}{49}$
g) $5x^4 - 80$	h) $(3x-4)^3 + (x+3)^3$	i) $(4x^2-4)^2 - 81x^4$
j) $a^4 - 16a^6b^2$	k) $a^6 + 7a^3 - 8$	L) $3a^4 - 15a^2 - 108$
m) $27x^3 + 64y^6$	n) $125x^3 - 8x^6$	o) $125x^6 + 64x^9$

2. For what value of "n" does $\left(2^{2007} - 2^{2006}\right)\left(2^{1997} - 2^{1996}\right) = 2^n$?

3. The number 2001 can be written as a difference of squares, $x^2 - y^2$ where "x" and "y" are positive integers in four different ways. What are the four possible ways?

4. Two numbers are such that their difference, their sum and their product are to one another as 1 : 7 : 18. The product of the two numbers are:

   a) 6 b) 12 c) 24 d) 48 e) none of these

5. The number 2005 can be written in the form of $a^2 - b^2$, where “a” and “b” are positive integers less than 1000 in exactly one way. What is the value of $a^2 + b^2$?

6. Solve for “a” and “b” in the expression: $1+\sqrt{8} = \left(a+\sqrt{2}\right)\left(b-\sqrt{2}\right)$

7. Given that “p” is a prime number, solve for “x”: $1000 - 8x^3 = 8\left(p\right)\left(4p-3\right)$

8. Evaluate : $\dfrac{2^{10}-1}{2^5-1}$

9. Suppose that $n^2 - 4 = 50\left(n-2\right)$ and “n” is not equal to 2. What is the value of “n”?

10. Find as many prime numbers “p” as you can so that the expression 5p+1 is a perfect square. How many prime numbers like this do you think there are? Prove that there are only this many primes.

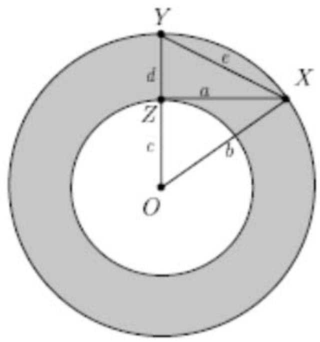
11. The positive difference of two perfect squares is 32. What is the largest possible value of the sum of the two perfect squares?

12. How many ordered pairs (m,n) of positive integers, with m>n, have the property that their squares differ by 96? List out all possible pairs.

13. Given that “p” is a prime number and the expression 2003p + 16 is a perfect square, what is the lowest possible value of “p”?

14. An annulus is the region between two concentric circles. The concentric circles in the figure have radii "b" and "c", with b > c. Let $\overline{OX}$ be a radius of the larger circle, let $\overline{XZ}$ be tangent to the smaller circle at Z, and let $\overline{OY}$ be the radius of the larger circle that contains Z. Let *a* = *XZ*, *d* = *YZ*, and *e* = *XY*. What is the area of the annulus?

(A) πa^2 **(B)** πb^2 **(C)** πc^2 **(D)** πd^2 **(E)** πe^2



15. If $a^2 - b^2 - 64$, what is the smallest possible values of $(a+b)$?

16. There are four different positive integers "a", "b", "c" , and "d" such that the equation is true: $a^3 + b^3 = c^3 + d^3 = 1729$. What is the values of *a* + *b* + *c* + *d*?

17. What is the sum of the (decimal) digits of $10^6 - 8$?

18. What are both pairs of integers (x,y) for which $4^y - 615 = x^2$

19. Show that if the positive integer "n" is a multiple of 3, then $7^n - 6^n$ is a multiple of 127.

20. Show that $2^{99}+3^{99}$ is divisible by 35.

21. Let “a” and “b” be the roots of the equation $x^2-mx+2=0$. Suppose that $a+\frac{1}{b}$ and $b+\frac{1}{a}$ are the roots of the equation: $x^2-px+q=0$. What is the value of q?

22. Challenge: March 2009 (Adler). Show that $n^{n-1}-1$ is divisible by $(n-1)^2$ for every positive integer “n”.

23. (Challenge) Let “x” and “y” be two-digit integers such that “y” is obtained by reversing the digits of “x”. The integers “x” and “y” satisfy $x^2-y^2=m^2$ for some positive integer “m”. What is the value of $x+y+m$?