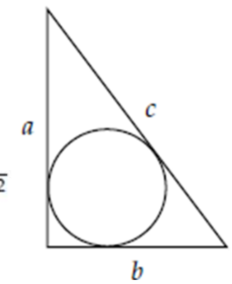
Name:__________________

5.6 Similar Triangles Part 2: Challenging Questions

1. A circle is inscribed in a right triangle with sides "a", "b", and "c" where "c" is the hypotenuse, as shown in the diagram. What is the radius of the circle?

(A) $\frac{1}{2}(a+b-c)$ (B) $\frac{1}{2}(a+b+c)$ (C) $\sqrt{a^2+b^2+c^2}$

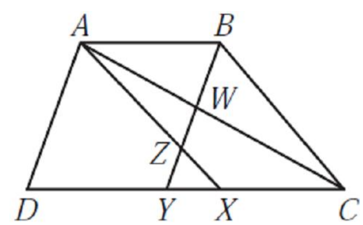
(D) $\frac{1}{2}\sqrt{a^2+b^2+c^2}$ (E) $a+b-c$



2. CNML 1979: In triangle ABC, AC=18, and "D" is the point on AC for which AD = 5. Perpendiculars drawn from "D" to AB and CD have lengths of 4 and 5 respectively. Find the area of triangle ABC.

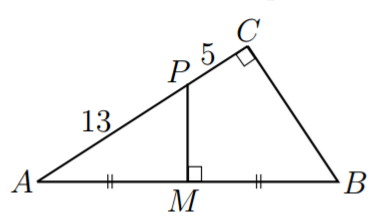
3. IN the diagram, ABCD is a trapezoid with AB parallel to CD and with AB=2, CD=5. Also , AX is parallel to BC and BY is parallel to AD. If AX and BY intersect at "Z", and AC and BY intersect at W, what is the ratio of the area of triangle AZW to the area of trapezoid ABCD is:

(A) 7 : 105 **(B)** 8 : 105 **(C)** 9 : 105
(D) 10 : 105 **(E)** 12 : 105



4. Triangle ABC is

Triangle ABC is right-angled at C, and $AC > BC$. The perpendicular bisector of the hypotenuse AB meets the hypotenuse at M and meets AC at P. Given that $AP = 13$ and $PC = 5$, what is the ratio of the area of $\triangle APM$ to the area of $\triangle ABC$? Express the answer as a common fraction.



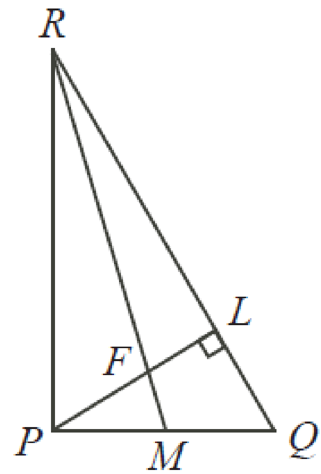
5. Let ABC be an equilateral triangle with sides of length 3. Let arc AC be the shorter circular arc with centre "B" joining "A" and "C", arc BC be the shorter circular arc with center "A" joining "B" and "C", and

$\overset{\frown}{AB}$ be the shorter circular arc with center "C" joining "A" and "B". See the diagram. What is the area of the shaded portion?

(A) $\frac{9}{4}(2\pi - 3\sqrt{3})$ (B) $\frac{9}{4}(2\pi - \sqrt{3})$ (C) $\frac{9}{2}\pi$

(D) $\frac{9}{2}(\pi + \sqrt{3})$ (E) $\frac{9}{2}(\pi - \sqrt{3})$

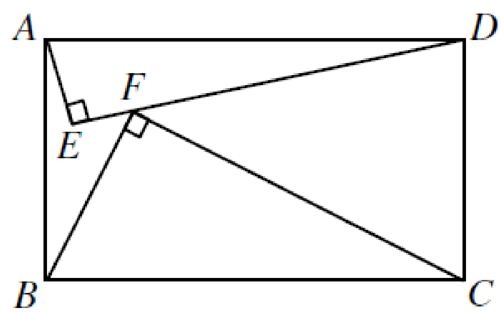
In the diagram, $\triangle PQR$ is right-angled at P and has $PQ = 2$ and $PR = 2\sqrt{3}$. Altitude PL intersects median RM at F. What is the length of PF?



(A) $\frac{\sqrt{3}}{2}$ **(B)** $\frac{3\sqrt{3}}{7}$ **(C)** $\frac{4\sqrt{3}}{7}$

(D) $\frac{5\sqrt{3}}{9}$ **(E)** $\frac{3\sqrt{3}}{5}$

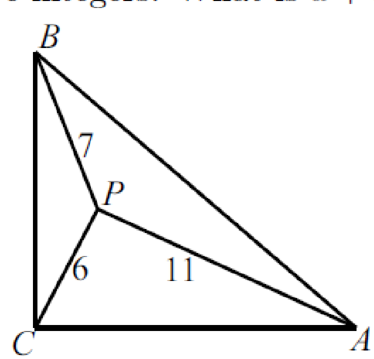
In the diagram, right-angled triangles AED and BFC are constructed inside rectangle $ABCD$ so that F lies on DE. If $AE = 21$, $ED = 72$ and $BF = 45$, what is the length of AB?



(A) 50 **(B)** 48 **(C)** 52

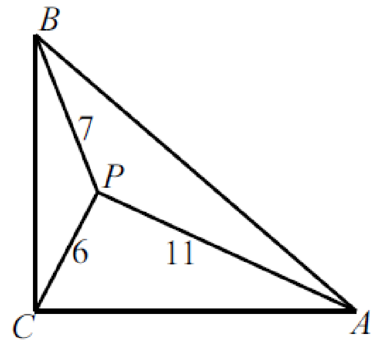
(D) 54 **(E)** 56

23. Isosceles $\triangle ABC$ has a right angle at C. Point P is inside $\triangle ABC$, such that $PA = 11$, $PB = 7$, and $PC = 6$. Legs $\overline{AC}$ and $\overline{BC}$ have length $s = \sqrt{a + b\sqrt{2}}$, where a and b are positive integers. What is $a + b$?



(A) 85 **(B)** 91 **(C)** 108 **(D)** 121 **(E)** 127

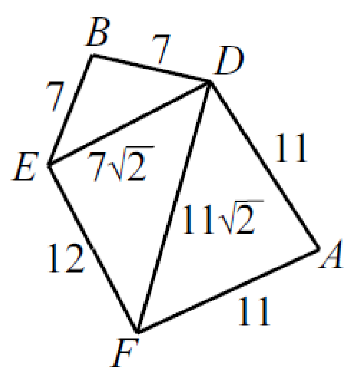
23. Isosceles $\triangle ABC$ has a right angle at C. Point P is inside $\triangle ABC$, such that $PA = 11$, $PB = 7$, and $PC = 6$. Legs $\overline{AC}$ and $\overline{BC}$ have length $s = \sqrt{a + b\sqrt{2}}$, where a and b are positive integers. What is $a + b$?



(A) 85 **(B)** 91 **(C)** 108 **(D)** 121 **(E)** 127

23. **(E)** Let D, E, and F be the reflections of P about $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively. Then $\angle FAD = \angle DBE = 90°$, and $\angle ECF = 180°$. Thus the area of pentagon $ADBEF$ is twice that of $\triangle ABC$, so it is s^2.

Observe that $DE = 7\sqrt{2}$, $EF = 12$, and $FD = 11\sqrt{2}$. Furthermore, $(7\sqrt{2})^2 + 12^2 = 98 + 144 = 242 = (11\sqrt{2})^2$, so $\triangle DEF$ is a right triangle. Thus the pentagon can be tiled with three right triangles, two of which are isosceles, as shown.

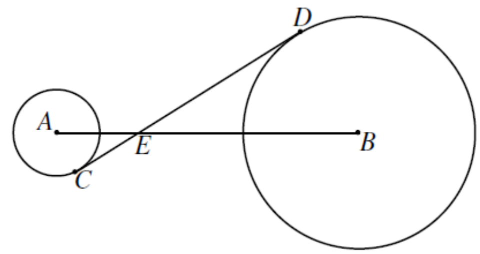


It follows that

$$s^2 = \frac{1}{2} \cdot (7^2 + 11^2) + \frac{1}{2} \cdot 12 \cdot 7\sqrt{2} = 85 + 42\sqrt{2},$$

so $a + b = 127$.

16. Circles with centers A and B have radii 3 and 8, respectively. A common internal tangent intersects the circles at C and D, respectively. Lines AB and CD intersect at E, and $AE = 5$. What is CD?



(A) 13 **(B)** $\frac{44}{3}$ **(C)** $\sqrt{221}$ **(D)** $\sqrt{255}$ **(E)** $\frac{55}{3}$

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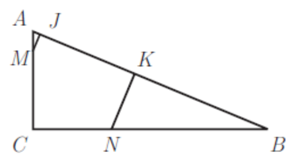
14. In $\triangle ABC$, $AB = 13$, $AC = 5$ and $BC = 12$. Points M and N lie on $\overline{AC}$ and $\overline{BC}$, respectively, with $CM = CN = 4$. Points J and K are on $\overline{AB}$ so that $\overline{MJ}$ and $\overline{NK}$ are perpendicular to $\overline{AB}$. What is the area of pentagon $CMJKN$?



(A) 15 **(B)** $\frac{81}{5}$ **(C)** $\frac{205}{12}$ **(D)** $\frac{240}{13}$ **(E)** 20

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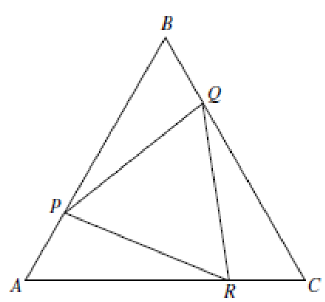
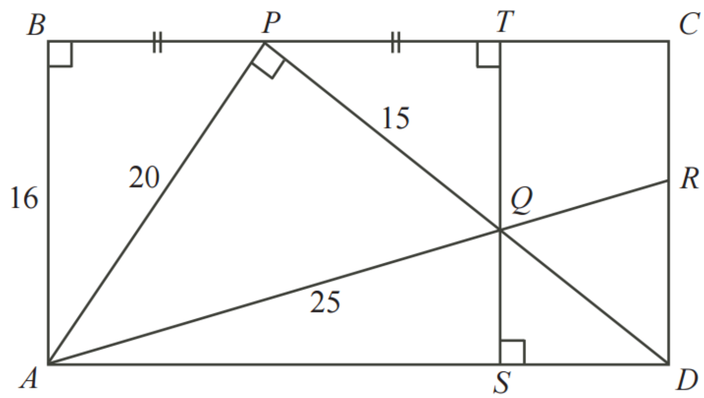
3. In the diagram, $\triangle ABC$ is equilateral with side length 4. Points P, Q and R are chosen on sides AB, BC and CA, respectively, such that $AP = BQ = CR = 1$.



(a) Determine the exact area of $\triangle ABC$. Explain how you got your answer.

(b) Determine the exact areas of $\triangle PBQ$ and $\triangle PQR$. Explain how you got your answers.

3. In rectangle $ABCD$, P is a point on BC so that $\angle APD = 90^\circ$. TS is perpendicular to BC with $BP = PT$, as shown. PD intersects TS at Q. Point R is on CD such that RA passes through Q. In $\triangle PQA$, $PA = 20$, $AQ = 25$ and $QP = 15$.



(a) Determine the lengths of BP and QT.

(b) Show that $\triangle PQT$ and $\triangle DQS$ are similar. That is, show that the corresponding angles in these two triangles are equal.

(c) Determine the lengths of QS and SD.

(d) Show that $QR = RD$.