## Math 11
### Chapter 2 – Trigonometry Review

Name: ___________________________ Solution ___________________________ Period: ___________

1. Determine two coterminal angles (one positive and one negative) and the reference angle for each given angle.

<table>
<thead>
<tr>
<th>a. $\theta = 290^\circ$</th>
<th>b. $\theta = -200^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 650^\circ$</td>
<td>$\theta = 160^\circ$</td>
</tr>
<tr>
<td>$\theta = -70^\circ$</td>
<td>$\theta = -560^\circ$</td>
</tr>
<tr>
<td>$\theta_r = 70^\circ$</td>
<td>$\theta_r = 20^\circ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c. $\theta = 520^\circ$</th>
<th>d. $\theta = -490^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 160^\circ$</td>
<td>$\theta = 230^\circ$</td>
</tr>
<tr>
<td>$\theta = -200^\circ$</td>
<td>$\theta = -130^\circ$</td>
</tr>
<tr>
<td>$\theta_r = 20^\circ$</td>
<td>$\theta_r = 50^\circ$</td>
</tr>
</tbody>
</table>

2. Determine the quadrant in which angle $x$ lies.

<table>
<thead>
<tr>
<th>a. $\sin x &lt; 0$ and $\tan x &gt; 0$</th>
<th>b. $\cos x &gt; 0$ and $\tan x &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quadrant III</strong></td>
<td><strong>Quadrant IV</strong></td>
</tr>
</tbody>
</table>

3. The point $P(-9, -12)$ is on the terminal arm of angle $\theta$. Find $\sin \theta$, $\cos \theta$, and $\tan \theta$.

$$(-9)^2 + (-12)^2 = r^2$$
$$81 + 144 = r^2$$
$$225 = r^2$$
$$r = 15$$

$$\sin \theta = \frac{-12}{15} = -\frac{4}{5}$$
$$\cos \theta = \frac{-9}{15} = -\frac{3}{5}$$
$$\tan \theta = \frac{-12}{9} = -\frac{4}{3}$$

4. If point $P$ is on the terminal arm in standard position making angle $\theta$, which is in the fourth quadrant, and $\sin \theta = -\frac{3}{\sqrt{10}}$. Determine the possible coordinates for $P$, and Find the other two trigonometric ratio for $\theta$.

$$x^2 + (-3)^2 = (\sqrt{10})^2$$
$$x^2 + 9 = 10$$
$$x = 1$$

$$\cos \theta = \frac{1}{\sqrt{10}}$$
$$\tan \theta = -\frac{3}{1}$$
5. Solve for the missing sides of the triangles. Express your answer in exact value.

\[ x = 6\sqrt{2} \]
\[ y = 12 \]

6. Find the values of \( \theta \) for \( 0^\circ < \theta \leq 360^\circ \).

\[ \sin \theta = -\frac{1}{\sqrt{2}} \]
Quadrant III: \( \theta = 225^\circ \)
Quadrant IV: \( \theta = 315^\circ \)
\[ \theta_r = 45^\circ \]

\[ \tan \theta = \sqrt{3} \]
Quadrant I: \( \theta = 60^\circ \)
Quadrant III: \( \theta = 240^\circ \)
\[ \theta_r = 60^\circ \]

7. Solve for \( \theta \) to the nearest degree, if \( 0^\circ \leq \theta < 360^\circ \).

\[ \cos \theta = -0.7515 \]
\( \theta = \cos^{-1}(0.7515) \)
Quadrant II: \( \theta = 139^\circ \)
Quadrant III: \( \theta = 221^\circ \)
\[ \theta = 41^\circ \]

\[ \tan \theta = -0.8642 \]
\( \theta = \tan^{-1}(0.8642) \)
Quadrant II: \( \theta = 139^\circ \)
Quadrant IV: \( \theta = 319^\circ \)
\[ \theta = 41^\circ \]

8. If the terminal arm of an angle \( \theta \), lies on the line \( 4x - 2y = 0 \), for \( x \leq 0 \), determine the exact value of \( \sin \theta + \cos \theta \).

\[ (1)^2 + (2)^2 = r^2 \]
\[ 1 + 4 = r^2 \]
\[ 5 = r \]

\[ \cos \theta = \frac{-1}{\sqrt{5}} \]
\[ \sin \theta = \frac{-2}{\sqrt{5}} \]
\[ \therefore \sin \theta + \cos \theta = \frac{-1}{\sqrt{5}} + \frac{-2}{\sqrt{5}} = \frac{-3}{\sqrt{5}} \]
9. Solve the triangle.

a. \[ R = 180 - (116 + 35.786^\circ) \]
   \[ R = 28.214^\circ \]

\[
\begin{align*}
\sin 116 &= \frac{\sin Q}{8.3} \\
5.4 \sin 116 &= \sin Q \\
35.786^\circ &= Q
\end{align*}
\]

\[
\frac{8.3}{\sin 116} = \frac{PQ}{\sin 28.214} \\
8.3 \sin 28.214 &= PQ \\
4.366 &= PQ
\]

b. \[ \angle R = 79^\circ \]

10. Solve for angle C.

a. \[ \angle A = 48^\circ, a = 4, \text{ and } c = 5 \]

\[
\begin{align*}
\sin C &= \frac{\sin 48}{5} \\
\sin C &= \frac{5 \sin 48}{4} \\
\angle C &= 68.269^\circ
\end{align*}
\]

\[ \angle C = 180 - 68.269^\circ \]

\[ \angle C = 111.731^\circ \]

- Ambiguous Case

b. \[ \angle A = 39^\circ, a = 28, \text{ and } c = 41 \]

\[
\begin{align*}
\sin C &= \frac{\sin 39}{41} \\
\sin C &= \frac{41 \sin 39}{28} \\
\angle C &= 67.147^\circ
\end{align*}
\]

\[ \angle C = 180 - 67.147^\circ \]

\[ \angle C = 112.853^\circ \]

- Ambiguous Case

11. An observer on the ground looks up to the top of a building at an angle of elevation of 30°. After moving 50 feet closer, the angle of elevation is now 40°. Find the height of the building.

\[
\frac{y}{\sin 140} = \frac{50}{\sin 10} \\
y = \frac{50 \sin 140}{\sin 10} \\
y = 185.08
\]

\[ \sin 30^\circ = \frac{h}{185.08} \]

\[ h = 185.08 \sin 30 \]

\[ h = 92.54 \text{ ft} \]
12. Solve the triangle.

(a) \[ A = 120^\circ, \quad B = 19, \quad C = 15 \]

\[ c^2 = 48^2 + 16^2 - 2(48)(16)\cos 115^\circ \]
\[ c^2 = 2560 - 1536\cos 115^\circ \]
\[ c = 56.65 \]

\[ \sin 115 = \frac{\sin A}{56.65} \]
\[ \frac{48\sin 115}{56.65} = \sin A \]
\[ \angle C = 180 - (115 + 50.17) \]
\[ 50.17^\circ = A \]
\[ \angle C = 14.83^\circ \]

(b) \[ A = 115^\circ, \quad B = 48, \quad C = 16 \]

\[ c = 56.65 \]

\[ \sin 115 = \frac{\sin A}{56.65} \]
\[ \frac{48\sin 115}{56.65} = \sin A \]
\[ \angle C = 180 - (115 + 50.17) \]
\[ 50.17^\circ = A \]
\[ \angle C = 14.83^\circ \]

(c) \[ f = 24.6, \quad d = 8.2, \quad e = 18.3 \]

\[ c^2 = 22.8^2 + 18.6^2 - 2(22.8)(18.6)\cos A \]
\[ 655.36 = 865.8 - 848.16\cos A \]
\[ -210.44 = -848.16\cos A \]
\[ 0.24811356 = \cos A \]
\[ 75.634^\circ = \angle A \]

\[ \sin 75.634 = \frac{\sin B}{25.6} \]
\[ \sin B = \frac{18.6\sin 75.634}{25.6} \]
\[ 44.736^\circ = \angle B \]

\[ \angle C = 180 - (75.634 + 44.736) \]
\[ \angle C = 59.63^\circ \]

(d) \[ a = 25.6, \quad b = 18.6 \]

\[ c = 22.8 \]

\[ \sin 75.634 = \frac{\sin B}{25.6} \]
\[ \sin B = \frac{18.6\sin 75.634}{25.6} \]
\[ 44.736^\circ = \angle B \]

\[ \angle C = 180 - (75.634 + 44.736) \]
\[ \angle C = 59.63^\circ \]
13. Two people started walking from the same point, at the same time; the walkers diverge at an angle of 110 degree. If one walks at rate 3.5 km/h and the other at 2.6 km/h. Find the distance between them after 4 hours.

\[ c^2 = 14^2 + 10.4^2 - 2(14)(10.4)\cos 110^\circ \]
\[ c^2 = 304.16 - 291.2\cos 110^\circ \]
\[ c = 20.09 \text{ km} \]

14. Two ships leave port at 4 p.m. One is headed at a bearing of N 38 E and is traveling at 11.5 miles per hour. The other is traveling 13 miles per hour at a bearing of S 47 E. How far apart are they when dinner is served at 6 p.m.?

\[ c^2 = 23^2 + 26^2 - 2(23)(26)\cos 95^\circ \]
\[ c^2 = 1205 - 1196\cos 95^\circ \]
\[ c = 36.18 \text{ miles} \]

15. A straight line tunnel is to be constructed through a mountain between points X and Y in the diagram below. Naturally, locations X and Y are not visible from each other, and with the mountain in the way, it is impossible to measure the distance between the two points. However, a third point, Z, is located from which both X and Y are visible, and for which the distances and angles indicated in the diagram are measured. Compute the required length of the tunnel.

\[ \sin 52.4^\circ = \frac{\sin Y}{621.3} \]
\[ \frac{455.8 \sin 52.4^\circ}{621.3} = \sin Y \]
\[ \frac{621.3}{35.538^\circ} = \angle Y \]
\[ \angle Z = 180^\circ - (52.4 + 35.538) \]
\[ \angle Z = 92.062^\circ \]

\[ \frac{621.3}{\sin 52.4^\circ} = \frac{XY}{\sin 92.062^\circ} \]
\[ \frac{621.3 \sin 92.062^\circ}{\sin 52.4^\circ} = XY \]

\[ 783.675 \text{ m} = \angle Y \]
16. Find the measure of length DE to the nearest unit.

\[ \angle \beta = 180 - 43.846 \]
\[ \angle \beta = 136.154^\circ \]
\[ c^2 = 15.7^2 + 8.93^2 - 2(15.7)(8.93)\cos 136.154^\circ \]
\[ c^2 = 326.2349 - 280.402 \cos 136.154^\circ \]
\[ c = 22.99 \]

\[ \sin \theta = \frac{\sin 115}{12} \]
\[ \sin \theta = \frac{12 \sin 115}{15.7} \]
\[ 43.846^\circ = \angle \theta \]
\[ \angle \alpha = 180 - (115 + 43.846) \]
\[ \angle \alpha = 21.154^\circ \]

\[ \frac{FG}{\sin 21.154} = \frac{15.7}{\sin 115} \]
\[ FG = \frac{15.7 \sin 21.154}{\sin 115} \]
\[ FG = 6.25 \]

\[ FD = 6.25 + 8.93 \]
\[ FD = 15.18 \]

17. Find the measure of angle A to the nearest tenth.

\[ \angle \alpha = 180 - 69 \]
\[ \angle \alpha = 111^\circ \]

\[ \sin A = \frac{\sin 111}{8.88} \]
\[ \sin A = \frac{8.88 \sin 111}{15.43} \]
\[ \angle A = 32.5^\circ \]