The problem with the General Form and the textbook method is twofold:

1. The function is not easily factorable
   \[ y = x^2 - 3x + 13 \]
2. The function has no x-intercepts
   \[ y = -x^2 - 4x - 7 \]

Completing the Square (CTS) is the second, and perhaps the best, method that addresses these two problems directly:

1. It changes any quadratic equation in general form to standard form

   \[
   \begin{align*}
   \text{General Form} & \quad y = ax^2 + bx + c \\
   \text{Standard Form} & \quad y = a(x - p)^2 + q
   \end{align*}
   \]

2. It makes graphing and finding the x-intercepts a lot easier and faster

*The CTS also “gives birth” to the Quadratic Formula, to help solve equations that cannot be factored, i.e., you get x-intercepts that are fractions or decimals

Let’s go back and ensure we understand what is a “perfect square trinomial”

- Notice that the constant term in the trinomial is always a perfect square
- Notice that in the trinomial, if you divide the coefficient of the second term by 2 and square the #, you get the constant term
- Notice that the constant term in the trinomial is always a positive value because of the squaring process

\[
\begin{align*}
   y &= (x+3)(x+3) = (x+3)^2 \\
   y &= x^2 + 6x + 9 \\
   \quad y &= (x+4)(x+4) = (x+4)^2 \\
   y &= x^2 + 8x + 16 \\
   \quad y &= (x-5)(x-5) = (x-5)^2 \\
   y &= x^2 - 10x + 25
\end{align*}
\]
Basically, what we’re doing is using CTS to create a “perfect square trinomial” to transform any quadratic equation from general form to standard form.

\[ y = ax^2 + bx + c \]
\[ y = x^2 + \frac{b}{a}x + \frac{c}{a} \]
\[ y = \left(x^2 + \frac{b}{a}x\right) + \frac{c}{a} \]

- Divide entire right side by ‘a’
- Put a set of bracket around the first two terms
- Divide \(\frac{b}{a}\) by 2, square the whole term, then add its opposite to the constant term \(\frac{c}{a}\)
- The trinomial inside the brackets is a perfect square, so simplify
- Eliminate the denominator, so multiply entire right side by ‘a’
- Recall that \(p = -\frac{b}{2a}\)
- Recall \(q = c - ap^2\), therefore we have \(q = c - a\left(-\frac{b}{2a}\right)^2 = q = c - \left(\frac{b^2}{4a}\right)\)
- Equation is now in standard form

**Example 1:** Given \(y = x^2 - 6x + 7\), complete the square and find the vertex, A of S, x & \(\gamma\)-intercepts, and the domain & range

\[ y = x^2 - 6x + 7 \]
\[ y = \left(x^2 - 6x\right) + 7 \]
\[ y = \left(x^2 - \left(6\right)x + \left(\frac{6}{2}\right)^2\right) + 7 + (-) \]
\[ y = \left(x^2 - x + \right) - \]
\[ y = (x - )^2 - \]
- Put brackets around first two terms
- Divide 2\(^{nd}\) term coefficient by __, ________ it, then add its _____ on the outside
- You now have a “_______ _________” trinomial
- Simplify to _________ form
• Vertex = __________
• A of S: \( x = \) __________
• \( y = ( -3)^2 - 2 \)
  \( y = ( \_\_\_\_)^2 - 2 \)
  \( y = \_\_\_\_ = \) __________
  \( (x-3)^2 - 2 \)
  \( (x-3)^2 \)
  \( \pm \sqrt{\_\_\_\_} = \sqrt{(x-3)^2} \)
  \( \pm \sqrt{\_\_\_\_} = x-3 \)
  \( x = \pm \sqrt{\_\_\_\_} \)
  \( x = +\sqrt{\_\_\_\_} \) and \( x = -\sqrt{\_\_\_\_} \)
  \( x = \_\_\_\_ \) and \( x = \_\_\_\_ \)

Domain: \{x\}  
Range: \{y\}  

Example 2: Given \( y = x^2 + 10x + 14 \), complete the square and find the vertex, A of S, \( x \) & \( y \)-intercepts, and the domain & range

• Remember the vertex is \((p,q)\)
• Remember that A of S is \( x = p \)
• For y-int, remember x-coordinate = ___

• For x-int, remember y-coordinate = ___

• Domain is all "x" that satisfy the function
• Range is all "y" that satisfy the function
Example 3: Given \( y = 2x^2 - 12x + 11 \), complete the square and find the vertex, \( A \) of \( S \), \( x \) & \( y \)-intercepts, and the domain & range

\[
y = 2x^2 - 12x + 11
\]

\[
y = \frac{2}{2} x^2 - \frac{12}{2} x + \frac{11}{2}
\]

\[
y = x^2 - 6x + \frac{11}{2}
\]

\[
y = \left( x^2 - x + \left( \frac{-6}{2} \right)^2 \right) + \frac{11}{2} - \left( \frac{-6}{2} \right)^2
\]

\[
y = \left( x - 3 \right)^2 - \frac{9}{2}
\]

- Divide entire right side by __
- Put bracket around 1st two terms
- Divide 2nd term coefficient by __, ______ it, then add its __________ on the outside
- Multiply entire right side by __
- Simplify perfect square trinomial into standard form

**Vertex:**

\[ A \text{ of } S: \ x = \]

**\( y \)-intercept:**

\[ x \text{-intercepts}: \]

**Domain:** \( \{ x | \} \)

**Range:** \( \{ y | \} \)