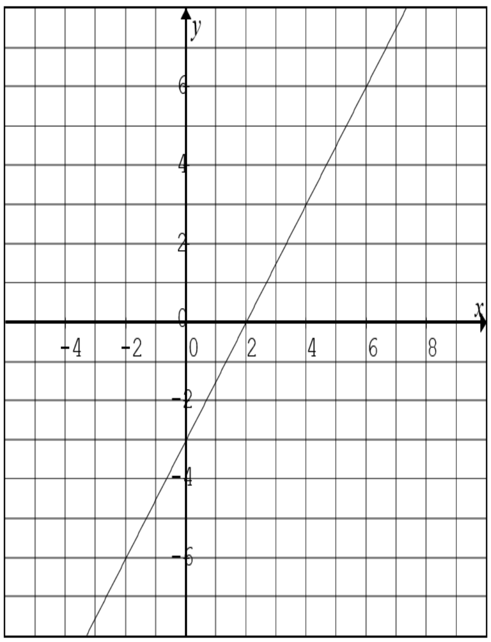
Precalculus 11 **Sec 3.1 – Quadratic Functions in Standard Form (Part 1)**

Let's do a quick review of <u>linear functions</u> from Pre-calc 10. The following diagram shows the graph of $3x - 2y = 6$.



- All linear functions can be expressed in standard form: $y = mx + b$
- $m = slope = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
- $b = \text{y-intercept } (0, y)$
- The degree of the equation is always 1
- Determine the equation of this function in standard form as well as the slope and y-intercept.

This year, we will look specifically at <u>quadratic functions</u>. They all have the following characteristics:

- The graph is "U"-shaped (aka, parabola)
- The equation in general form is: $y = Ax^2 + Bx + C$, where A, B, & C are real numbers
- The degree of the equation is always 2

Let's assume $A = 1$, $B = 0$, & $C = 0$. If we substitute these numbers into the general form equation

$y = Ax^2 + Bx + C$

$y = 1x^2 + 0x + 0$ this will be our "standard" quadratic equation

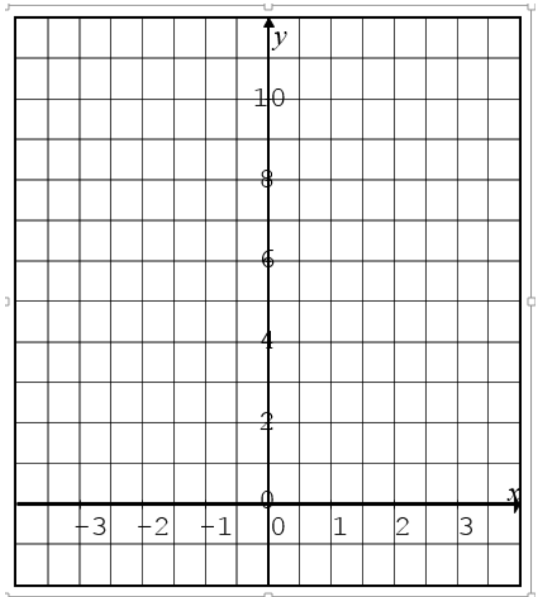
$y = x^2$

Example 1: Determine A, B, & C for each equation

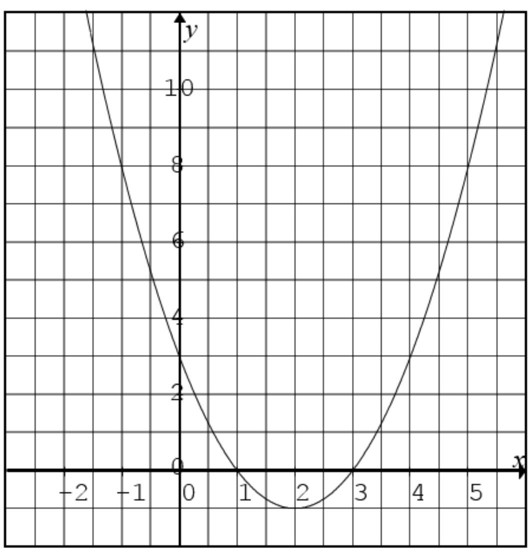
a) $y = 3x^2 + 2x - 5$	b) $y = 5x^2 + 17$	c) $y = 6x + 12$

We're going to use a table of values and the "standard" equation $y = x^2$ to see why quadratic functions have parabolic, ie, "U"-shaped graphs.

x	y
-3	
-2	
-1	
0	
1	
2	
3	



Let's look at specific terms and properties of quadratic functions using the equation $y = x^2 - 4x + 3$ and it corresponding graph.



- Vertex: The tip of the parabola (min or max) Our graph shows a min point @ $(2, -1)$
- Axis of Symmetry: A line cutting the graph in half, resulting in mirror images. Our graph shows an A of S @ $x = 2$
- X-intercepts: point(s) where the graph touches/crosses the horizontal axis. Our graph shows x-intercepts @ $x = 1$ & $x = 3$ or @ $(1, 0)$ & $(3, 0)$
- Y-intercept: point where the graph crosses the vertical axis. Our graph has a y-intercept @ $y = 3$ or @ $(0, 3)$

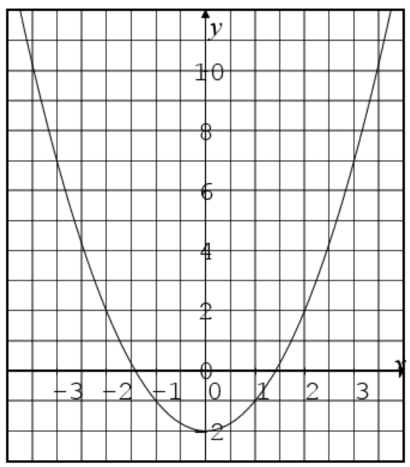
These four terms will be very important for you to understand.

Don't forget the following when determining your intercepts:

- To <u>calculate the x-intercepts</u>, remember the <u>y-coordinate = 0</u>
- To <u>calculate the y-intercepts</u>, remember the <u>x-coordinate = 0</u>

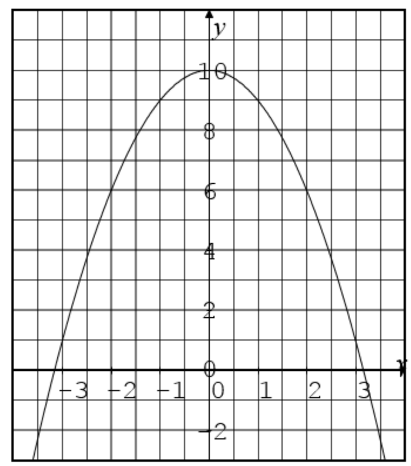
If we look at 2 quadratic functions with the equations $y = x^2 - 2$ & $y = -x^2 + 10$ and their corresponding graphs, please take notice of a few more things.

$y = x^2 - 2$



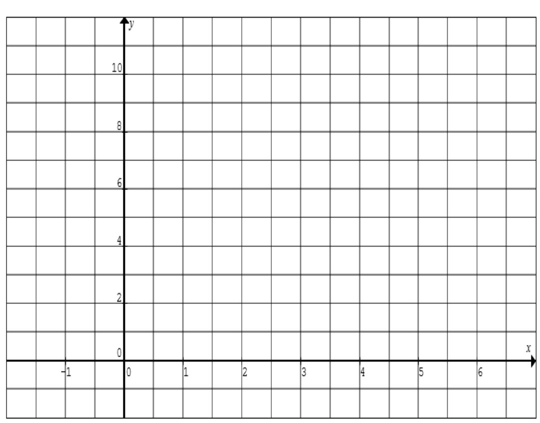
- The graph "opens up"
- The vertex $(0,-2)$ is a "minimum pt."
- The domain is $x \in \mathbb{R}$
- The range is all values above the vertex, i.e., $y \geq -2$

$y = -x^2 + 10$



- The graph "opens down"
- The vertex $(0,10)$ is a "maximum pt."
- The domain is $x \in \mathbb{R}$
- The range is all values below the vertex, i.e., $y \leq 10$

Example 2: Graph the function $y = x^2 - 5x + 6$ and answer the following questions.



x	-1	0	1	2	3	4	5	6
y								

a) Vertex:

b) A of S:

c) x-intercepts:

d) y-intercept:

e) Domain:

f) Range:

It will be especially important for us to be able to express quadratic functions in Standard Form.

- Using a table of values will enable you to draw the graph, but it takes too long and is inefficient
- The standard form for a Quadratic Function is: $\boxed{y = a(x-p)^2 + q}$
- These are the following characteristics:

1. Vertex: (p,q) 2. A of S: $x = p$ 3. x-intercept: make $y = 0$ and solve for x	4. y-intercept: make $x = 0$ and solve for y 5. Domain: $x \in \mathbb{R}$ 6. Range: $y \le q$ or $y \ge q$

Example 3: Determine a, p, & q, vertex, and A of S for each equation.

a) $y = (x-3)^2 + 4$	b) $y = -\frac{1}{2}(x+4)^2 - 11$	c) $y = 12 - 6(x+9)^2$

The 'a' value in $y = ax^2$ serves two functions:

1. It will determine whether the graph opens up or down, i.e., is there a reflection about the horizontal axis
   - If $a = (+)ve$, the graph will open up
   - If $a = (-)ve$, the graph will open down, i.e., reflected about the x-axis
2. It will determine whether the graph is tall & thin or short & fat, i.e., is there a vertical expansion/compression
   - If $a \ge 1$, the graph will become tall & thin, i.e., vertical expansion
   - If $0 \le a \le 1$, the graph will become short & fat, i.e., vertical compression

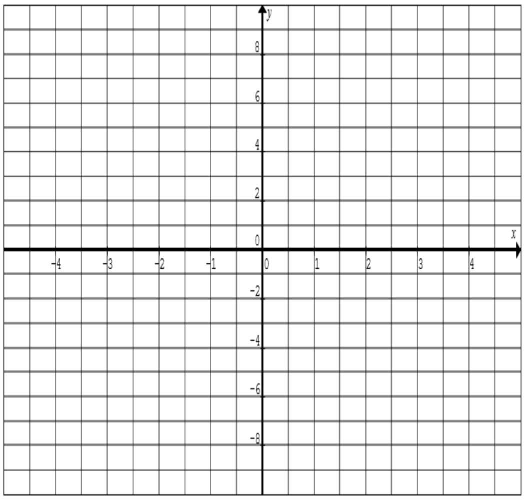
The most important fact is that only the y-coordinates are affected through multiplication

- Any vertical expansion or compression will change the shape of the graph
  - Shape of new graph will NOT BE CONGRUENT to original graph

- Any vertical reflection will not change the shape of the graph
    - Shape of new graph will BE CONGRUENT to original graph

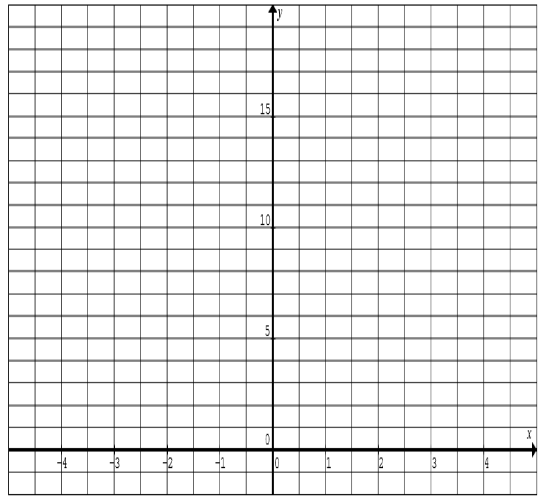
Example 4: Graph the following quadratic functions.

a) Given $y = x^2$, graph $y = -x^2$



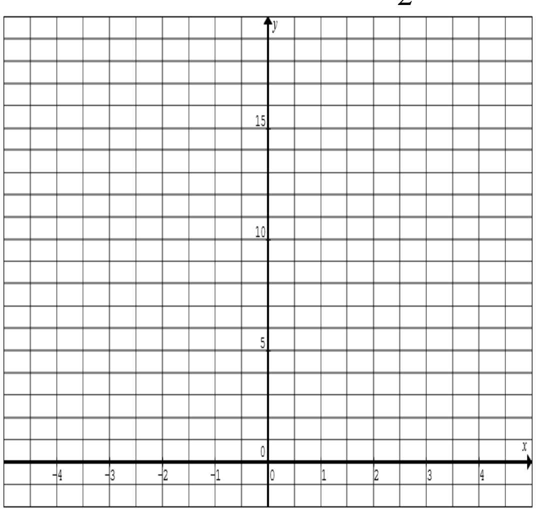
x							
y							
y'							

b) Given $y = x^2$, graph $y = 2x^2$



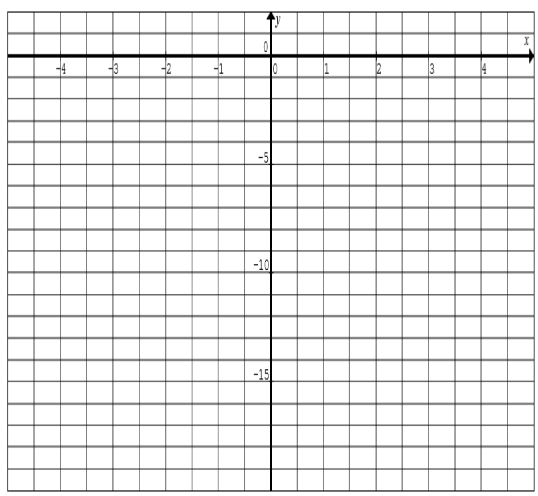
x							
y							
y'							

c) Given $y = x^2$, graph $y = \frac{1}{2}x^2$



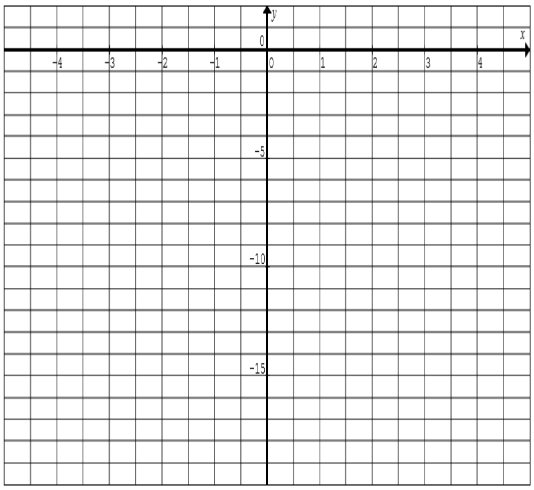
x							
y							
y'							

d) Given $y = -x^2$, graph $y = -4x^2$



x							
y							
y'							

e) Given $y=-x^2$, graph $y=-\frac{1}{3}x^2$



x							
y							
y′							