Precalculus 11 **Sec 2.3x – The Sine Law (Ambiguous Case)**

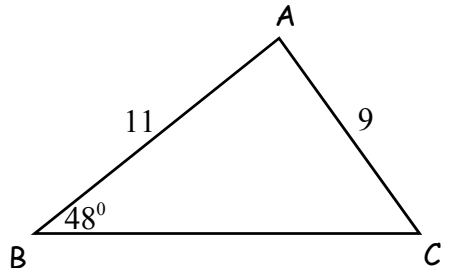
So far, all the triangles we've seen that uses the sine law had a unique solution.

The **ambiguous case** occurs when we are given 2 sides and an angle that is not contained by the 2 sides (SSA), ie, we have 2 sides and an angle opposite one of those sides.

- This will result in the triangle having
  - no solution
  - exactly one solution
  - two possible solutions

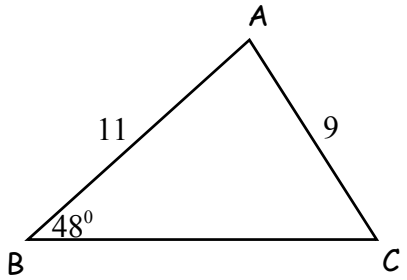
The following example shows how you will get 2 possible solutions.

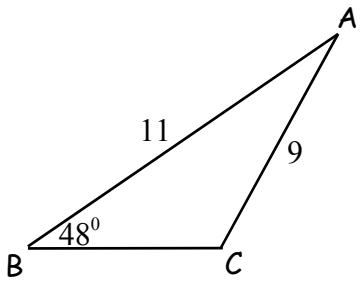
Solve ΔABC, given $\angle B = 48^0$, b = 9, and c = 11



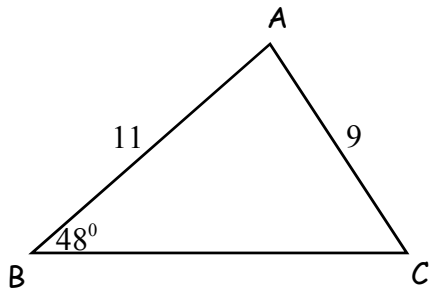
Draw an altitude and calculate the height (h)

Since $b = 9$, and is bigger than $h = 8.17$, one triangle can be acute and the other can be oblique/obtuse, that is they will look like the following:

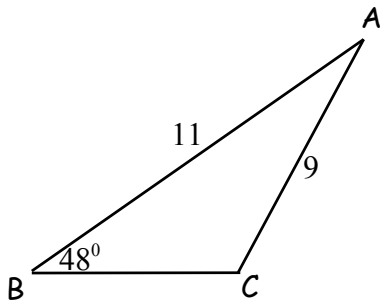




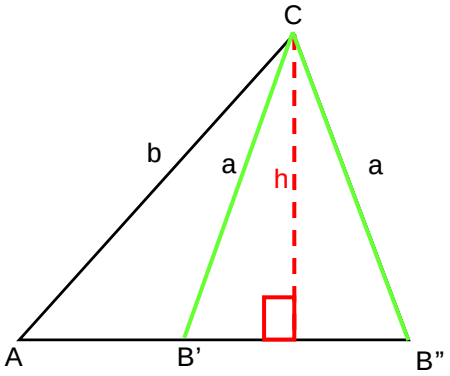
Solving the acute triangle gives us 1 of 2 possible solutions for the original triangle.



Solving the obtuse triangle will give the 2nd solution for the original triangle. The key is to get the obtuse angle by subtracting the calculated acute angle from 180^0.



In summary, given we are given $\angle A$ and sides 'a' and 'b' for any ΔABC, there will always be <u>2 possible solutions, if and only if</u>

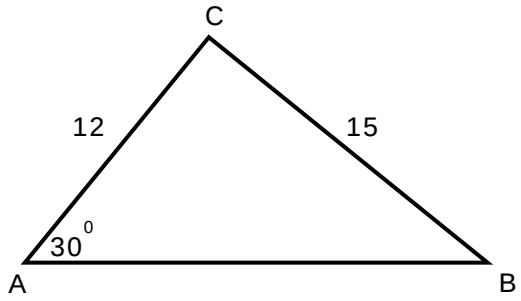


If $h = b\sin A$, then $h < a < b$, which is the same as $b\sin A < a < b$

The following two examples shows how you will only get <u>1 solution</u>.

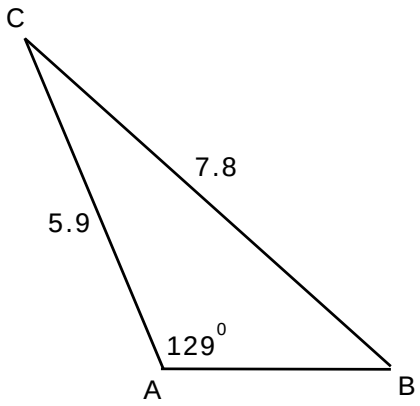
Solve ΔABC, given $\angle A = 30^0$, a = 15, and b = 12

Construct an altitude and find the height (h)



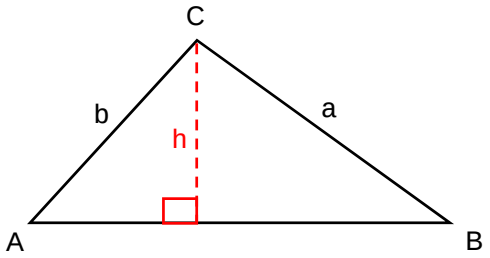
Since $a = 15 > b = 12$, which is $> h = 6$, only one triangle is possible. Solve.

Solve ΔABC if $\angle A = 129^0$, a = 7.8, and b = 5.9

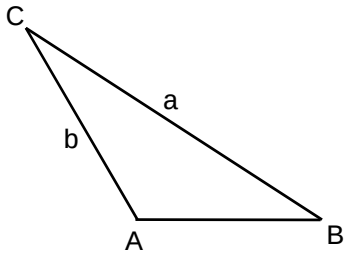


Since $a = 7.8 > b = 5.9$, only one triangle will be possible. Solve.

In summary, if we are given $\angle A$ and sides 'a' and 'b' for any ΔABC, there will always be <u>1 solution, if and only if</u>



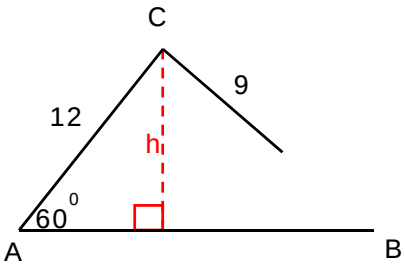
If $h = b\sin A$, then 1) $b > h$ & $a = h$ which is the same thing as $a = b\sin A$ or 2) $b > h$ & $a \geq b$



In this case, 'h' is considered irrelevant as long as $a > b$

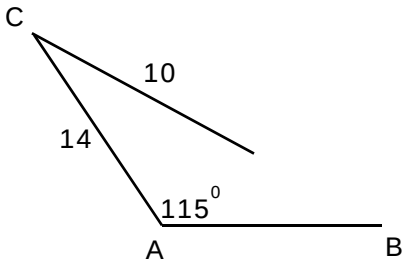
There is <u>no solution</u> if any of the following situations occur

Solve ΔABC, given $\angle A = 60^0$, a = 9 & b = 12



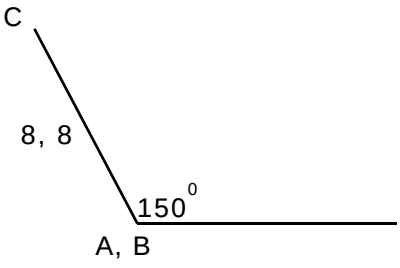
$a < h$, or $a < b\sin A$

Solve ΔABC, given $\angle A = 115^0$, a = 10 & b = 14



$a < b$

Solve ΔABC, given $\angle A = 150^0$, a = 8 & b = 8



$a = b$

Example: Justin and Jayden are holding the lines to their kites flying up in the sky. Justin's line is 25m long at an angle of inclination of 45^0. Jayden's line is 20m long. To the nearest meter, what is the distance between the two boys?

Homework: