Precalculus 11

Sec 2.2 - Trigonometric Ratios of any Angle

Just to reiterate and reinforce, if we are given any right triangle we can always use the two following concepts to solve the triangle:

Find the length of any side using Pythagoras if 2 sides are known

\[ \text{hyp} = \sqrt{\text{opp}^2 + \text{adj}^2} \]

Find the length or angle of any triangle using basic trig ratios

\[
\begin{align*}
\sin \theta &= \frac{\text{opp}}{\text{hyp}} \\
\cos \theta &= \frac{\text{adj}}{\text{hyp}} \\
\tan \theta &= \frac{\text{opp}}{\text{adj}}
\end{align*}
\]

Just a few important concepts to remember from section 2.1:

- The “unit circle” has a radius (terminal arm) equal to 1
- The tip of the terminal arm always has coordinates of (x, y)
- A right triangle can be constructed using the terminal arm, and the sum of 'x^2' and 'y^2' will equal 1 unit

The radius of any unit circle is always equal to 1 unit

\[ x^2 + y^2 = r^2 \]

This is also the formula for a circle with the center located at the origin (0, 0)

Therefore, given any angle in standard position, you can always find the coordinates of any point on the unit circle using trigonometry (r = 1)

\[
\begin{align*}
\sin \theta &= \frac{y}{r} \\
\cos \theta &= \frac{x}{r} \\
\tan \theta &= \frac{y}{x}
\end{align*}
\]

Since \( r = 1 \), this means \( \sin \theta = y \) and \( \cos \theta = x \), thus the coordinates of any point P on the unit circle is represented by \( P(x, y) = P(\cos \theta, \sin \theta) \)

Example 1: Determine the coordinates of a point \( P(x, y) \) on a unit circle, given the angles in standard position.

a) \( \theta = 215^\circ \)

b) \( \theta = 150^\circ \)
The sin, cos, tan of angles in any quadrant will be either (+)ve or (-)ve

- Use the reference angle to determine whether the ratios are (+)ve or (-)ve

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
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<tbody>
<tr>
<td>( \sin \theta = \frac{\text{opp}}{\text{hyp}} )</td>
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<td>( \cos \theta = \frac{\text{adj}}{\text{hyp}} )</td>
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- This is also known as the CAST rule. Each letter represents the trig ratio that will be positive in each quadrant.

**Example 2:** Given the following trig ratios, determine which quadrant(s) in which the angle is located.

a) \( \sin \theta = -\frac{2}{3} \)

\[ \begin{array}{c}
\text{Q1} \\
\text{Q2} \\
\text{Q3} \\
\text{Q4}
\end{array} \]

b) \( \cos \theta = \frac{2}{\sqrt{5}} \)

c) \( \tan \theta = -\frac{3}{\sqrt{7}} \)

**Example 3:** Find the angle \( \theta \), given the following trigonometric function.

d) \( \sin \theta = \frac{1}{\sqrt{3}} \)

e) \( \cos \theta = -\frac{3}{5} \)

\[ \begin{array}{c}
\text{Q1} \\
\text{Q2} \\
\text{Q3} \\
\text{Q4}
\end{array} \]

f) \( \tan \theta = -\frac{2}{\sqrt{5}} \)
Given any trig ratio, you can find the coordinates of any endpoint on the terminal arm of a unit circle.

- **Method 1**: find the angle in standard position and use reference angles to find the other angle and its corresponding coordinates.
  - **Example 4**: Given the following trig ratios, find all possible coordinates for a point P(x, y) on the unit circle.
    
    a) \( \sin \theta = -\frac{5}{13} \)  
    b) \( \tan \theta = -\frac{1}{2} \)

- **Method 2**: find exact values by creating right triangles and using Pythagoras, then use the values to find the sides of the other trig ratios.
  - **Example 5**: Given the following trig ratios, find the exact coordinates of a point P(x, y) on the unit circle.
    
    c) \( \tan \theta = -\frac{\sqrt{11}}{5} \)  
    d) \( \cos \theta = -\frac{2}{7} \)
Example 4: Point $Q(-12, -9)$ is on the terminal arm of a circle in standard position.

- Give exact coordinates of a point $P(x, y)$ on the terminal arm of a unit circle
- What is the value of $\tan \theta$ and the measure of angle $\theta$?

Homework: