An infinite geometric series is any geometric sequence that has an infinite number of terms.

- If the common ratio is greater than 1 \((r > 1)\) or less than \(-1 \((r < -1)\), each term in the series becomes larger in either direction and the sum of the series gets closer to infinity, making it impossible to find a sum.
  - For example, there’s no way to determine the sum of this geometric sequence: \(3 + 6 + 12 + 24 + \ldots + 3145728 + 6291456 + \ldots\) because the numbers are getting way too big.

- To find a sum, the common ratio has to be between \(-1 \& 1\) \((-1 < r < 1)\), meaning each term in the series gets smaller and smaller that they eventually become insignificant; like adding zeros.
  - For instance, we can find the sum of the following geometric sequence: \(1 + 0.5 + 0.25 + 0.125 + \ldots + 0.0000000476 + 0.000000238 + \ldots\) because the terms are getting super small.

To summarize,

- If \(r > 1\), the sum of an infinite series cannot be determine because it diverges towards \(+\infty\)
- If \(r < -1\), the sum of an infinite series cannot be determined because it diverges towards \(-\infty\)
- If \(-1 < r < 1\), the sum of any infinite geometric series can be obtained because the series eventually converges to a fixed value.

**Example 1**: What should the common ratio be so that the following infinite geometric series, \(a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = 25\), converges to 25?

<table>
<thead>
<tr>
<th>a</th>
<th>(r = 2)</th>
<th>b</th>
<th>(r = \frac{7}{6})</th>
<th>c</th>
<th>(r = 0.80)</th>
<th>d</th>
<th>(r = 1.25)</th>
</tr>
</thead>
</table>

Use the formula for a finite series and apply it to an infinite series when \(-1 < r < 1\),

\[
S_n = \frac{a(1-r^n)}{(1-r)}
\]

\[
S_n = \frac{a(1-r^n)}{(1-r)} = \frac{a(1-0)}{(1-r)} = S_n = \frac{a}{1-r}
\]

- Since \(-1 < r < 1\), and \(n\) is infinite, \(r^n = 0\)
  - As an example: \((0.5)^{10} = 0.000976\)
  - \((0.5)^{30} = 0.0000000000000000931\)
- Our finite formula is now a formula for any infinite geometric series
Example 2: Find the sum of the following infinite geometric series

\begin{align*}
\text{a)} & \quad 14 + 7 + 3.5 + 1.75 + \ldots \\
\text{b)} & \quad 18 + 12 + 8 + \frac{16}{3} + \ldots
\end{align*}

Example 3: The common ratio of an infinite geometric series is 0.75. If the sum of all the terms converges to 20, what is the 1st term?

Example 4: A particular movie generated revenue of $2,500,000 in its opening week. Each week, revenue drops by 6%. If this particular movie is shown for a very long time, what is the total possible revenue generated?

Example 5: A ball is dropped from a height of 12 feet and bounces to 70% of its original height for each subsequent bounce. What is the total vertical distance the ball traveled when it finally comes to rest?

Homework: