An infinite geometric series is any geometric sequence that has an infinite number of terms.

- If the common ratio is greater than 1, ($r > 1$) or less than -1 ($r < -1$), each term in the series becomes larger in either direction and the sum of the series gets closer to infinity, making it impossible to find a sum.
  - For example, there's no way to determine the sum of this geometric sequence: $3+6+12+24+\ ...\ +3145728+6291456+\ ...$ because the numbers are getting way too big.
- To find a sum, the common ratio has to be between -1 & 1, ($-1 < r < 1$), meaning each term in the series gets smaller and smaller that they eventually become insignificant; like adding zeros
  - For instance, we can find the sum of the following geometric sequence: $1+0.5+0.25+0.125+\ ...\ +0.000000476+0.000000238+\ ...$ because the terms are getting super small

To summarize,

- If $r > 1$, the sum of an infinite series cannot be determine because it diverges towards +∞
- If $r < -1$, the sum of an infinite series cannot be determined because it diverges towards -∞
- If $-1 < r < 1$, the sum of any infinite geometric series can be obtained because the series eventually converges to a fixed value.

Example 1: What should the common ratio be so that the following infinite geometric series, $a + ar + ar^2 + ar^3 + \ \cdots\ + ar^{n-1} = 25$, converges to 25?

a) $r = 2$	b) $r = \frac{7}{6}$	c) $r = 0.80$	d) $r = 1.25$

Use the formula for a finite series and apply it to an infinite series when $-1 < r < 1$,

$S_n = \frac{a(1-r^n)}{(1-r)}$ $S_n = \frac{a(1-r^n)}{(1-r)} = \frac{a(1-0)}{(1-r)}$ = $\boxed{S_n = \frac{a}{1-r}}$	Since $-1 < r < 1$, and n is infinite, $r^n = 0$ As an example: $(0.5)^{10} = 0.000976$ $(0.5)^{30} = 0.000000000931$ Our finite formula is now a formula for any infinite geometric series

Example 2: Find the sum of the following infinite geometric series

a) $14+7+3.5+1.75+\ ...$	b) $18+12+8+\frac{16}{3}+\ ...$

Example 3: The common ratio of an infinite geometric series is 0.75. If the sum of all the terms converges to 20, what is the 1st term?

Example 4: A particular movie generated revenue of $2 500 000 in its opening week. Each week, revenue drops by 6%. If this particular movie is shown for a very long time, what is the total possible revenue generated?

Example 5: A ball is dropped from a height of 12 feet and bounces to 70% of its original height for each subsequent bounce. What is the total vertical distance the ball traveled when it finally comes to rest?

Homework: