What is a geometric series?

- A geometric series is a sequence in which the terms are summed together.
  - The terms are separated by a "+" sign instead of a comma.

Remember: Arithmetic and geometric sequences are different in that:

- Arithmetic sequences ADD/SUB a constant (common difference, 'd')
- Geometric sequences MULT/DIV by a factor (common ratio, 'r')

Example 1: Find the sum for each geometric series.

a. $5+12.5+31.25+78.125+195.3125$	b. $120+60+30+15+7.5+3.75+1.875+0.9375$

The quicker way to get the answer is to look for patterns. Consider the following:

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \ldots + ar^{n-1}$$

$$r \times \left(S_n = a + ar + ar^2 + ar^3 + ar^4 + \ldots + ar^{n-1}\right)$$
$$rS_n = ar + ar^2 + ar^3 + ar^4 + \ldots + ar^{n-1} + ar^n$$

$$\begin{aligned} S_n &= a + ar + ar^2 + ar^3 + ar^4 + \ldots + ar^{n-1} \\ -(rS_n &= \quad ar + ar^2 + ar^3 + ar^4 + \ldots + ar^{n-1} + ar^n) \end{aligned}$$
$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$\boxed{S_n = \frac{a(1-r^n)}{(1-r)}}$$

- This shows the sum for a finite/exact number of terms.
- Multiply whole equation by 'r' since each term increases by a common ratio, hence increasing the sum by a factor of 'r' also.
- This 2nd equation is a variation of the 1st equation, so we simplify by subtracting the 2 equations.
- Factor out common terms on both sides of the equation.
- Isolate S_n and you have the formula to find the sum of any finite geometric series

Example 2: Determine the sum of the following geometric series.

a) $9+18+36+72+144+288+576+1152$	b) $14+7+3.5+1.75+0.875+0.4375+0.21875$

Example 3: The sum of the first 10 terms of a geometric series is -29 524. What is the 5^{th} term if the common ratio is -3?

Example 4: Find the sum of: $6.5+26+104+\ldots+436207616$.

Example 5: A person is taking tetracycline for a chest infection. Each tablet contains 200 mg and about 12% of the mass remains in the body when the next tablet is taken. What is the mass of tetracycline in the body after 12 tablets?

Example 6: A ball is dropped from a height of 10 m. After each bounce it returns to 65% of its previous height. Determine the total vertical distance just before the 8^{th} bounce.

Example 7: The sum of the 2^{nd} and 3^{rd} term in a geometric series is 45. The sum of the 4^{th} and 5^{th} term is 20. Determine the geometric sequence.

Homework: